How Spin Affects Handbags: An Exploration of the Handbag Model

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Abstract

After establishing and deriving some conventions and foundational concepts of particle physics, I calculate Feynman diagrams for both weak and electromagnetic interactions in order to investigate the effects on spin. I then relate these effects to the parton model.
Acknowledgements

I cannot thank Professor McAskill enough for all of her patience, guidance, and chocolate during the course of this project. I could not have asked for a better thesis (and life) advisor.

Thank you also to Professor Hu for your endless positivity (and hugs), and to Professor Miller for your technological wisdom.

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Chapter 1:

Introduction

Particles are the fundamental building blocks of matter. The development of particle physics is often remembered as a “succession of brilliant insights and heroic triumphs” by individuals rather than as the clever guesswork and collaborative editing process that it truly was [1]. As a result, in this discussion of the evolution of particle physics within the 20th century I hope to emphasize that these great achievements and useful results, at least those of the Klein-Gordon equation and Dirac equation, were a (not-so linear) combination of genius guesswork and endless revision.

Schrödinger and Relativity

In the academic wilderness, when a physics student encounters microscopic objects, they hopefully realize that this belongs within the realm of quantum mechanics. Further, when the word particle is thrown around or when any concept of particle physics is first introduced, their initial instinct might be that the behavior of the particle is governed by the quantum mechanics of Schrödinger:

\[ i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle. \]  

Indeed, Schrödinger’s formulation of quantum mechanics, or more specifically the Schrödinger wave equation (1), is useful in describing particle behavior at non-relativistic speeds. However, as a particle’s speed approaches the speed of light, the rules of special relativity insist that, in order for a formulation to accurately describe a relativistic object’s behavior, it must transform appropriately from frame to frame. Because Schrödinger’s equation, as it stands,

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1 While there are many small objects that the word particle can describe, particle physics is usually concerned with describing the tiniest indivisible particle able to be detected.

2 Especially those at a subatomic level.
does not transform according to these rules, it is not a Lorentz invariant quantity.

Recall that the rules of special relativity ensure that the laws of physics apply in any inertial reference frame; this is Einstein’s first postulate of relativity. The second postulate of special relativity states that the speed of light, $c$, in a vacuum is the same in all reference frames. While some of the trademark features of special relativity are time dilation, length contraction, and the relativity of simultaneity, what leads to the correct relativistic mathematical formulation of particle physics are Lorentz transformations.

Lorentz transformations, or equivalently Lorentz transforms, are linear transformations between inertial frames that are in constant relative motion. To represent these frame transformations more neatly a new quantity called a four-vector is introduced, $x^\mu$, which tells us how the space and time components of an object relate to one another:

$$x^\mu = (x^0, x^1, x^2, x^3) = (ct, x, y, z).$$

This quantity, $x^\mu$, is the position four-vector, and because it has an upper index it is also known as a contravariant four-vector. Furthermore, it is possible to express this position four-vector with a lower index, $x_\mu$, which is known as a covariant four-vector. This switch does require some careful sign accounting though:

$$x_\mu = (x_0, x_1, x_2, x_3) = (ct, -x^1, -x^2, -x^3)$$

and so the space components of $x_\mu$ are the negative of those of $x^\mu$.

How would one switch between these two representations, or rather, how does one raise or lower an index? The Minkowski metric, $g^{\mu\nu}$, does just the trick:

$$g^{\mu\nu} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix} = g_{\mu\nu}. \quad (4)$$

Therefore, another way to represent $x_\mu$ is $x_\mu = g_{\mu\nu}x^\nu$.

More generally, four-vectors are vectors with one time-like component, $x_0$, and three space-like components, $x_1, x_2, \text{ and } x_3$. While non-relativistic formulations often have time and space on different orders (and thus they are not treated equally), this notation solves this problem; space and time are on the same order, exactly as relativity requires.

Additionally, because of its structure, when a proper four-vector is contracted with itself, it produces an invariant scalar, meaning that from frame to frame, the result of this contraction is unchanged.\(^3\) However, it is true that for

\[^3\text{Proper meaning Lorentz invariant.}\]
any two proper four-vectors, like $a^\mu$ and $b^\mu$ for example, the quantity

$$a_\mu b^\mu = a^\mu b_\mu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3 = a^0 b^0 - a \cdot b$$

(5)

is invariant; a contraction between any two four-vectors produces an invariant quantity. This is the four-vector equivalent to the dot product\(^4\) For better or for worse though, there is no four-vector cross product equivalent.\(^5\)

In addition, recall from earlier that part of the usefulness of four-vectors is their ability to put space and time on the same order, which is required in order for equations to be Lorentz invariant. If we rewrite (1) as

$$\left( -\frac{\hbar^2}{2m} \nabla^2 + V(x) \right) \psi(r,t) = i\hbar \frac{\partial}{\partial t} \psi(r,t),$$

it is clear that space and time are on different orders. This demonstrates that Schrödinger’s equation would never be Lorentz invariant at relativistic speeds.

**Relativistic Schrödinger Equation Attempt One: the Klein-Gordon Equation**

So, how do we get a relativistic wave equation? If the Schrodinger equation can be derived from the classical energy-momentum relation

$$\frac{p^2}{2m} + V(x) = E$$

(6)

then this seems like a good place to start. Using Einstein’s energy relation from special relativity yields

$$E^2 - p^2 c^2 = m^2 c^4$$

(7)

which can be rewritten with four-vector notation as

$$p^\mu p_\mu - m^2 c^2 = 0,$$

(8)

where $p^\mu$ is the momentum four-vector:

---

\(^4\)I’ll use bold text for our good old three-vectors.

\(^5\)There is something close, but the result is actually a tensor, not a four-vector. Additionally, for those who are curious, we classify a scalar as a tensor of rank 0, vector as a tensor of rank 1, whereas a second-rank tensor is a step up from four-vectors in that it has two indices rather than one. For more information on tensors and mathematical four-vector stuff, check out chapter 3 of Griffith’s *Introduction to Elementary Particles.*
\[ p^\mu = \left( \frac{E}{c}, p \right) = -i\hbar \partial^\mu. \] (9)

In (6), where \( p \) and \( E \) would be replaced with the operators

\[ \hat{\rho} = -i\hbar \nabla \] (10)

and

\[ \hat{H} = i\hbar \frac{\partial}{\partial t} \] (11)

respectively, similar substitutions can be made in (8) for our momentum four-vector:

\[ p_\mu = i\hbar \partial_\mu, \] (12)

where \( \partial_\mu \) is

\[ \partial_\mu = \left( \frac{\partial}{\partial t}, \nabla \right) = \left( \frac{1}{c} \frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right). \] (13)

Accounting for all of these changes and substituting them into (8) yields

\[ -\hbar^2 \partial^\mu \partial_\mu \psi - m^2 c^2 \psi = 0 \] (14)

or, with some more adjusting

\[ -\frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} + \nabla^2 \psi = \left( \frac{mc}{\hbar} \right)^2 \psi. \] (15)

This is the Klein-Gordon equation, which applies to bosons. It was initially unpopular among prominent quantum physicists of the time, such as Schrödinger and Born, because it could not reproduce Bohr’s energy levels for the hydrogen atom. It was also incompatible with Max Born’s statistical interpretation of quantum mechanics.

### Relativistic Schrödinger Equation Attempt Two: the Dirac Equation

Historically, the next challenge was to find an equation that was not only first order in time but also still consistent with (7). Dirac picked up the gauntlet; he attempted to factor (8), though in order to do this more easily, we set \( p = 0. \)
This leaves only the $p^0$ of $p^\mu$:

$$(p^0)^2 - m^2 c^2 = (p^0 + mc)(p^0 - mc) = 0.$$  

Now, there are two first-order equations $(p^0 - mc) = 0$ and $(p^0 + mc) = 0$ that satisfy (8). Once the spatial terms contained in $p$ are included, though, these equations are of the form:

$$p^\mu p_\mu - m^2 c^2 = (\beta^\kappa p_\kappa + mc)(\gamma^\lambda p_\lambda - mc)$$

where $\beta^\kappa$ and $\gamma^\lambda$ are as-yet unknown coefficients, and, expanding the right hand side produces:

$$\beta^\kappa \gamma^\lambda p_\kappa p_\lambda - mc(\beta^\kappa - \gamma^\kappa)p_\kappa - m^2 c^2.$$  

To free $p_\kappa$ from having linear terms so that we can end up with the relativistic energy conservation formula in the end, we require $\beta^\kappa = \gamma^\kappa$. Additionally, $\gamma^\kappa$ must allow:

$$p^\mu p_\mu = \gamma^\kappa \gamma^\lambda p_\kappa p_\lambda.$$  

If these $\gamma$'s are numbers, there is no way to get rid of all of the cross terms that appear when this equation is expanded (see Griffiths *Introduction to Elementary Particles*, page 228). What could they be?

Dirac realized that they must be matrices, and unsurprisingly these gammas are known as the gamma matrices:

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \gamma^1 = \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix}, \gamma^2 = \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix}, \gamma^3 = \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix},$$  

where $\sigma_1$, $\sigma_2$, and $\sigma_3$, are the Pauli spin matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. $$  

There is an additional gamma matrix called $\gamma^5$ where

$$\gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} = i\gamma^0 \gamma^1 \gamma^2 \gamma^3.$$  

We will discuss the properties of these matrices in the next chapter. But, let us return to (7), which has evolved into the form
\[(p_\mu p_\mu - m^2 c^2) = (\gamma^\mu p_\mu - mc) (\gamma^\lambda p_\lambda + mc) = 0.\] 

(19)

It is conventional to choose the first term, \((\gamma^\mu p_\mu - mc)\), to get the Dirac Equation, as well as to substitute \(p_\mu\) with (12):

\[(i\hbar \gamma^\mu \partial_\mu - mc)u(\vec{p}) = 0\] 

(20)

Some further conventions insist that we set \(\hbar = c = 1\), and so (20) is more often shown as

\[(i\gamma^\mu \partial_\mu - m)u(\vec{p}) = 0.\] 

(21)

For more on unit conventions of particle physics, see Table 1 below.

<table>
<thead>
<tr>
<th>SI Units</th>
<th>Natural Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>SI units are good for ‘everyday’ physics. Typically, the base units of SI units are mass (m), length (x), and time (t), which we use to derive other quantities.</td>
<td>Natural units are good for particle physics. While the base units of natural units are energy (E), velocity (v), and angular momentum (L), it is much more useful to think of energy and momentum (p) instead once we follow convention and set (\hbar = c = 1):</td>
</tr>
<tr>
<td>([m] = \text{kg})</td>
<td>([E] = \text{GeV})</td>
</tr>
<tr>
<td>([x] = \text{m})</td>
<td>([v] = c = 1)</td>
</tr>
<tr>
<td>([t] = \text{s})</td>
<td>([L] = \hbar = 1)</td>
</tr>
<tr>
<td></td>
<td>([p] = \text{GeV})</td>
</tr>
<tr>
<td></td>
<td>The SI basis units (mass, length, and time) now become derived quantities:</td>
</tr>
<tr>
<td></td>
<td>([m] = \text{GeV})</td>
</tr>
<tr>
<td></td>
<td>([x] = \text{GeV}^{-1})</td>
</tr>
<tr>
<td></td>
<td>([t] = \text{GeV}^{-1})</td>
</tr>
</tbody>
</table>

Table 1: The SI units most students are familiar with compared to the natural units of particle physics.

**Solutions to the Dirac Equation: The Dirac Spinor**

\(u(\vec{p})\) is known as a Dirac spinor and is a column vector with four components:
\[ u(p) = \sqrt{E + m} \left( \begin{array}{c} u_A \\ \frac{\sigma \cdot p}{(E + m)} u_A \end{array} \right) = \sqrt{E + m} \left( \begin{array}{c} u_A \\ u_B \end{array} \right) \]  \hspace{1cm} (22)

where:

\[ u_A = \left( \begin{array}{c} u_1 \\ u_2 \end{array} \right), \quad u_B = \frac{\sigma \cdot p}{(E + m)} \left( \begin{array}{c} u_1 \\ u_2 \end{array} \right). \]  \hspace{1cm} (23)

The values one would insert for \( u_1 \) and \( u_2 \) change depending upon the spin of the particle. Generally, for a particle that has a spin with an angle \( \theta \), spin up is

\[ u_A = \left( \begin{array}{c} \cos \left( \frac{\theta}{2} \right) \\ e^{-i\phi} \sin \left( \frac{\theta}{2} \right) \end{array} \right), \]  \hspace{1cm} (24)

whereas spin down is

\[ u_A = \left( \begin{array}{c} -e^{-i\phi} \sin \left( \frac{\theta}{2} \right) \\ \cos \left( \frac{\theta}{2} \right) \end{array} \right). \]  \hspace{1cm} (25)

If, on the other hand, the particle’s spin has no angle, these take the familiar forms

\[ u_A = \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \]  \hspace{1cm} (26)

\[ u_A = \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \]  \hspace{1cm} (27)

for spin up and spin down, respectively.

The Dirac equation (21) and its spinor (22) describe the behavior of fermions. Before diving deeper into solutions of the Dirac equation and bilinear covariants though, as we will in the next chapter, a brief summary is in order. In both the derivations of the Klein-Gordon equation and the Dirac equation, we started with the energy and momentum relation formula. The reason that the Klein-Gordon equation was initially viewed as insufficient (or as a mistake), was because it is second order in time, meaning that it could not reproduce or follow certain expected results of non-relativistic quantum mechanics. Dirac, however, successfully produced an equation that was first order in both time and space; in order to do so, the introduction of gamma matrices along with some accompanying mathematical machinery became necessary.

Moving on, there are a few more conventions to address, continuing with the solutions to the Dirac equation. There are four solutions total, two of which are associated with particles. Let’s suppose that a particle is at rest. The
Dirac equation then becomes:

\[ i\hbar \gamma^0 \frac{\partial \psi}{\partial t} - mc \psi = 0 \]

or

\[
\begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}
\begin{pmatrix}
\frac{\partial \psi_A}{\partial t} \\
\frac{\partial \psi_B}{\partial t}
\end{pmatrix}
= -i \frac{mc^2}{\hbar}
\begin{pmatrix}
\psi_A \\
\psi_B
\end{pmatrix}
\]

where

\[
\psi_A = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}
, \quad
\psi_B = \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix}
\]

so

\[
\frac{\partial \psi_A}{\partial t} = -i \frac{mc^2}{\hbar} \psi_A,
\frac{\partial \psi_B}{\partial t} = +i \frac{mc^2}{\hbar} \psi_B.
\]

Therefore, the solutions to this equation are

\[
\psi_A(t) = e^{-i \frac{mc^2}{\hbar} t} \psi_A(0),
\psi_B(t) = e^{+i \frac{mc^2}{\hbar} t} \psi_B(0).
\]

Now, recall that \( e^{-i \frac{mc^2}{\hbar} t} = e^{\frac{-E}{\hbar} t} \). This is the characteristic time dependence of a quantum state of energy \( E \), where \( E = mc^2 \) when a particle is at rest. So, while \( \psi_A(t) \) is of a form we would expect when a particle is at rest, \( \psi_B(t) \) ends up disappointing us because it gives a negative energy, \( E = -mc^2 \). Well, can’t we just avoid this mess by saying that \( \psi_B(t) = 0 \)? Unfortunately not; since the positive energy states alone are not complete, and because “in a quantum system we need a complete set of states”, we must keep these negative energy states [1]. While Dirac tried to avoid this problem by postulating the existence of an “unseen infinite ’sea’ of negative-energy particles, which fill up all those unwanted states”, convention now dictates that \( \psi_B(t) \) represents antiparticles with positive energy [1].

**Feynman Diagram Introduction**

There is another topic we should consider before diving into the kinematics of particle interactions: Feynman diagrams. The most efficient way to describe an interaction is through Feynman diagrams. If one has ever encountered such a figure before, one would note that there are striking similarities between the diagram and children’s scribbles. Unlike these doodles, however, each piece of a Feynman diagram is imbued with meaning, and all necessary bits are compiled

---

8 There are some elaborate derivations, which one could find in a standard particle physics textbook, that extend the solution we found for a particle at rest to the more general plane-wave solutions of a particle (or antiparticle) in motion. This textbook would ideally also contain more information on antiparticles, though it is up to the curious student to seek it out.
What’s Next?

Now that we are equipped with the proper tools to study relativistic quantum mechanics, it is time to see these principles in action. While chapter 2 will detail the kinematics of the interaction of interest, chapters 3 and 4 have the herculean task of applying and synthesizing all of the information presented in chapter 1. In particular, we will see how the interactions presented in chapters 2 and 3 play into the parton model.

Figure 1: The components of Feynman diagrams, as well as their multiplicative factors [2].

neatly in Figure 1.
Chapter 2: Kinematics

Feynman Diagrams: the S, T, and U Channels

When reading Feynman diagrams, no matter their orientation, time always flows from left to right. Thus, if we follow the conventions of time in Figure 2, we see that the interaction starts on the left with the aptly-named incoming fermion on the bottom left and the (massive) boson, produced by the interaction on the top left, on the top. It is necessary to check that at each vertex quantities such as spin, flavor, lepton number and charge are conserved (which they are, in our case). Flowing from left to right, the internal leg of this Feynman diagram, or fermion propagator, shows how an
interaction occurred; its type (i.e. fermion, boson, etc.) is determined by conservation rules. Moving further still, the fermion propagator leg flows into the meson, a particle made by a quark and antiquark pair, which then flows into the outgoing fermion.

While Figure 2 will remain in a fixed orientation, in general the orientation of a Feynman diagram can be changed in order to observe another representation of the same interaction. Each orientation is called a channel, of which there are three in total: s, t, and u, though the s channel representation is the most common. As with many things in particle physics, switching channels requires some careful accounting, namely by checking that conservation laws still hold in the new figure. Further, a given interaction needs at most two diagrams to give us enough information to explore a process in depth and to give an electromagnetically invariant quantity, but it turns out anyway that not all three channels are allowed at once [3].

![Feynman diagram]

Figure 3: The s, t, and u channels; they all represent the same kinematic interaction but in different ways.

But what are s, t, and u? Are they only convenient naming tools, or do they hold physical significance? As Figure 3 suggests, these quantities are combinations of the four-momenta of the propagating particles, where:
\[
\begin{align*}
  s &= (p_A + p_B)^\mu (p_A + p_B)_\mu = (E_A + E_B)^2 - (p_A + p_B)^2, \\
  t &= (p_A - p_C)^\mu (p_A - p_C)_\mu = (E_A - E_C)^2 - (p_A - p_C)^2, \\
  u &= (p_A - p_D)^\mu (p_A - p_D)_\mu = (E_A - E_D)^2 - (p_A - p_D)^2.
\end{align*}
\]

At each vertex of a Feynman diagram four-momentum is conserved, but these quantities are more than a neat way of keeping track of them all; these allow us to get around the fact that we can not talk about synchronicity and simultaneity in special relativity, but we can talk about invariant intervals instead.

**Kinematics of the Weak Interaction**

With all of this in mind, let’s return to Figure 2 and begin by analyzing the incoming quantities on the left. The incoming (quark-type) fermion travels in the \(+z\) direction with four-momentum \(k^\mu = (k_0, 0, 0, k_3)\). We are restricting the kinematics of this interaction to a single plane, and so we use (26) and (27) to represent this particle’s spin up and down respectively so that both the spin and momentum are aligned along the \(z\)-axis. This also influences how we represent the spin of the outgoing (quark-type) fermion, whose four-momentum is given by \(p^\mu = (p_0, p_1, 0, p_3)\); we set \(\phi = 0\), so in (24) and (25) the exponential term can be dropped, leaving us with

\[
\begin{align*}
  u_A &= \begin{pmatrix}
  \cos \left( \frac{\theta}{2} \right) \\
  \sin \left( \frac{\theta}{2} \right)
\end{pmatrix},
\end{align*}
\]

for spin up and

\[
\begin{align*}
  u_A &= \begin{pmatrix}
  -\sin \left( \frac{\theta}{2} \right) \\
  \cos \left( \frac{\theta}{2} \right)
\end{pmatrix},
\end{align*}
\]

for spin down.

The incoming boson, generated by the interaction at the top left, is traveling in the \(−z\) direction with four-momentum \(k'^\mu = (k_0', 0, 0, -k_3')\), and because this interaction occurs in the center of mass (CoM) frame, \(k + k' = 0\). The boson’s polarization, \(\varepsilon^\mu\), is either transverse (for spin = \(±1\)), where the components parallel to its momentum are \(\varepsilon_0 = \varepsilon_3 = 0\):
\[ \epsilon^\mu = \frac{\mp \epsilon_1 - i \epsilon_2}{\sqrt{2}} = \frac{1}{\sqrt{2}} (0, \mp 1, -i, 0), \]

or a longitudinal (for spin = 0) polarization, where the components perpendicular to its momentum are \( \epsilon_1 = \epsilon_2 = 0: \)

\[ \epsilon^\mu = \frac{1}{M} \left( k'_3, 0, 0, k'_0 \right). \]

Additionally, \( k'^\mu \epsilon_\mu = 0, \) and for transverse polarizations \( \epsilon^\mu \epsilon_\mu = -1 \) while for longitudinal polarizations \( \epsilon^\mu \epsilon_\mu = +1. \)

It is also worthwhile to note the relevant values of \( s, t, \) and \( u \) for this interaction:

\[ s = (k + k')^\mu (k + k')_\mu = (k + k')^2, \]

\[ t = (p - k)^\mu (p - k)_\mu = (p - k)^2, \]

and

\[ u = (p - q)^\mu (p - q)_\mu = (p - q)^2. \]

Finally,

\[ s + t + u = m_1^2 + m_2^2 + m_3^2 - Q^2. \]

Now that we have made note of these important kinematic quantities, we will see how, from the Feynman diagram pictured in Figure 2, they come together to form invariant equations known as amplitudes. Generally, these equations begin to bridge the gap between theoretical work and experimental results, as well as reveal the underlying mechanisms responsible for observed behaviors in a given interaction.
Chapter 3: Calculation Details

The Weak Interaction

Before getting into the messy calculation details, there is one question left to tackle: what is the weak interaction? The weak interaction, also known as the weak force, is one of the four fundamental forces. It involves the exchange of the massive intermediate vector bosons $W^+$, $W^-$, referred to as charged current, and $Z^0$, referred to as neutral current. But why is the weak force weak? The massive sizes of the $W$ and $Z$ bosons ($80\text{ GeV}$ and $91\text{ GeV}$ respectively) are largely to blame. Unlike interactions mediated by massless particles, which have an infinite range, because of the sizes of these bosons the weak force is only effective over short distances ($10^{-18}\text{ m}$). Further, such a large size requires a lot of energy to produce them, and thus these interactions are rare.

While only particles with color can participate in the strong interaction, and while only charged particles can interact electromagnetically, all fermions (both quarks and leptons) can interact weakly. As interesting as the apparent inclusivity of the weak force is, what makes it more interesting is its ability to change the flavor or type of one particle into another, as well as the essential role it plays in decay processes, crucial ones which enable us to live. This flavor changing ability allows us to study a wider variety of particles (as well as their underlying structure) in a given interaction.

Spin and Amplitude Equations for the Weak Interaction

However, in order to see other properties and preferences of the weak force, we must look at conserved quantities, namely spin. Fortunately for us, there is no reason why conservation rules should not hold at relativistic speeds.

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9 Compare these to the size of a proton, which is roughly $0.9\text{ GeV}$. Because we are using natural units (shown in how we set $\hbar = c = 1$), units of mass and momentum are represented with units of energy, generally in $eV$, because $E^2 = p^2 + m^2$.

10 In particular, leptons ($e^-$, $\mu^-$, $\tau^-$, and their associated neutrinos) do not participate in strong interactions because they lack a property known as color, and neutrinos also do not interact electromagnetically because they have no charge.

11 Without it, the sun would have no deuterium for fusion to take place, for example.
One quantity that remains conserved is angular momentum. I suspected that this conservation rule would manifest in a spin preference since spin is a type of angular momentum. That is why, for the weak interaction pictured in Figure 2, I analyzed the different spin orientation combinations of the incoming and outgoing fermions in order to find the system’s spin preference. It turns out that this spin preference is governed by terms in the associated amplitude equation.

Recall from Figure 1 that a Feynman diagram is more than a handy visual tool; each piece contains information that helps us form what is called an amplitude equation. The amplitude equation for the interaction in Figure 2 is given by

\[
\bar{u}(p) \gamma^5 \frac{i(q^\nu q_\nu + m)}{(q^\mu q_\mu - m^2)} \left( -iC\gamma^\mu \frac{1}{2} \left( c_v - \gamma^\nu c_a \right) \epsilon_\mu \right) u(k) \tag{28}
\]

where \(\bar{u}(p) = u(p)^\dagger \gamma^\mu\) and represents the outgoing fermion spinor while \(u(k)\) is the spinor of the incoming fermion. While the spin orientation is not specified in (28), I later assigned spin orientations to the incoming and outgoing fermion spinors in this equation in a program called Maple so that I could study different spin combinations of this system. For the transverse polarization, the resulting equations are compiled neatly in Table 2 (for clarity, the denominator term in (28) is left off of both amplitude equation tables).

<table>
<thead>
<tr>
<th>Spin Orientation</th>
<th>Amplitude Equations for Weak Interaction</th>
<th>Preferred or Not Preferred</th>
</tr>
</thead>
</table>
| Both spin up                     | \[
\frac{1}{\sqrt{k_0 + m_1 \sqrt{p_0 + m_2}}} (-\epsilon_1 + i\epsilon_2) \left\{ c_v \left[ -\sin \left( \frac{\theta}{2} \right) \left\{ \left( -q_0 + m_3 \right) \left( -p_0 - m_2 \right) k_3 - p \left( k_0 + m_1 \right) \left( q_0 + m_3 \right) \right\} \right\] + c_v \left[ -\sin \left( \frac{\theta}{2} \right) \left\{ \left( -p_0 + m_3 \right) \left( -q_0 - m_2 \right) \left( k_0 + m_1 \right) \right\} \right]\right\} \right\} \] | Not Preferred |
| Incoming spin up/Outgoing spin down (Spin Flip) | \[
\frac{1}{\sqrt{k_0 + m_1 \sqrt{p_0 + m_2}}} (-\epsilon_1 + i\epsilon_2) \left\{ c_v \left[ \cos \left( \frac{\theta}{2} \right) \left\{ \left( q_0 + m_3 \right) \left( -p_0 - m_2 \right) k_3 + p \left( k_0 + m_1 \right) \left( q_0 + m_3 \right) \right\} \right\] + c_v \left[ \cos \left( \frac{\theta}{2} \right) \left\{ -p k_3 \left( q_0 + m_3 \right) + \left( p_0 + m_2 \right) \left( -q_0 - m_3 \right) \left( k_0 + m_1 \right) \right\} \right]\right\} \right\} \] | Preferred |
| Incoming spin down/Outgoing spin up (Spin Flip) | \[
\frac{1}{\sqrt{k_0 + m_1 \sqrt{p_0 + m_2}}} (-\epsilon_1 + i\epsilon_2) \left\{ c_v \left[ \cos \left( \frac{\theta}{2} \right) \left\{ \left( q_0 - m_3 \right) \left( p_0 + m_2 \right) k_3 + p \left( k_0 + m_1 \right) \left( q_0 + m_3 \right) \right\} \right\] + c_v \left[ \cos \left( \frac{\theta}{2} \right) \left\{ p k_3 \left( q_0 + m_3 \right) + \left( p_0 + m_2 \right) \left( q_0 - m_3 \right) \left( k_0 + m_1 \right) \right\} \right]\right\} \right\} \] | Preferred |
| Both spin down                   | \[
\frac{1}{\sqrt{k_0 + m_1 \sqrt{p_0 + m_2}}} (-\epsilon_1 + i\epsilon_2) \left\{ c_v \left[ \sin \left( \frac{\theta}{2} \right) \left\{ \left( -q_0 + m_3 \right) \left( -p_0 + m_2 \right) k_3 - p \left( k_0 + m_1 \right) \left( q_0 + m_3 \right) \right\} \right\] + c_v \left[ -\sin \left( \frac{\theta}{2} \right) \left\{ p k_3 \left( q_0 + m_3 \right) + \left( p_0 + m_2 \right) \left( q_0 - m_3 \right) \left( k_0 + m_1 \right) \right\} \right]\right\} \right\} \] | Not Preferred |

Table 2: Weak amplitudes and preferences.
Collapsing the Amplitude: Finding the Controlling Dirac Bilinear for the Weak Interaction

While one could later use this amplitude equation to obtain a cross section equation, we see that “the physics resides” primarily in the invariant amplitude, and thus it is more fruitful for us to analyze (28) [2]. In this case, analyze means expand the amplitude equation in terms of Dirac bilinears. All Feynman diagrams and their amplitudes can be collapsed into one primary Dirac bilinear which describes the most prominent or preferred behavior of a system. For (28), the result of collapsing (leaving out the propagator denominator for clarity) looks like:

$$\frac{C}{2} \bar{u}(p) \left\{ e_{\mu} p_{\nu} \left( \gamma^5 g^{\mu\nu} c_v + i \gamma^5 \sigma^{\mu\nu} c_v - g^{\mu\nu} c_a - i \sigma^{\mu\nu} c_a \right) + m e_{\mu} \left( - \gamma^5 \gamma^5 c_v + \gamma^5 c_a \right) \right\} u(k).$$  (29)

Before anything else, it is worth noting that we are considering the ultra-relativistic limit; essentially, this means that we ignore any mass terms in (29). From there, there are a few ways to figure out which operator is in control for (29). The first would be to pull out your favorite particle physics textbook and analyze the functions of each of the operators. After a few hours of study, one would eventually be able to conclude that the terms containing the operator $g^{\mu\nu}$, like $\gamma^5 g^{\mu\nu} c_v$ and $-g^{\mu\nu} c_a$, do not matter since they will contract with the four-vectors and thus give us $\bar{u}(p)u(k)$. That leaves us with the terms $-i \sigma^{\mu\nu} c_a$ and $+i \gamma^5 \sigma^{\mu\nu} c_v$. Thus, $\sigma^{\mu\nu}$ is the major controlling operator for spin preference, and due to its structure, it prefers a spin flip. The second method, which involves using Maple to compare the four amplitudes in Table 2 with the spin combinations fused with different Dirac bilinears, works well alone or with the first, as it confirms that $\sigma^{\mu\nu}$ is indeed in charge.

While $\sigma^{\mu\nu}$ is the controlling operator for this weak interaction, we will see that this is not the case for similar electromagnetic interactions. Therefore, while the weak interaction of Figure 2 has a spin flip preference, the electromagnetic interaction will not necessarily share this. Before diving into the next set of calculation details, though, it would be beneficial to pose the following questions: Why might such a comparison be done? What is the significance of these differences in spin preference? In order to answer these questions and thus to see the bigger picture (as well as the reason for this study) we will have to look at the parton model, which is discussed in full in the following chapter.
The Electromagnetic Interaction, or DVCS

Figure 4: S-channel Feynman diagram for DVCS. Notice the symmetry! This will be reflected in our amplitude equations.

Figure 4 shows an electromagnetic interaction known as Deeply Virtually Compton Scattering or DVCS. It is worth noting again that electromagnetic interactions cannot change the flavor or a particle, and thus we cannot study as wide a variety of particles as we can in weak interactions. However, they can still teach us about the structure of particles they interact with.

Deep Inelastic Scattering (DIS)

In order to understand how, let’s consider Deep Inelastic Scattering (DIS). This occurs when a probe (for us, this is a boson in both cases) has just enough energy to resolve information about the structure of a composite particle and to interact with one of its constituent quarks (known as partons) through hard (or elastic) scattering, without exciting its internal structure. DIS can sometimes result in a shower of particles if the probe excites the internal structure of a composite particle. This shower is known as a hadronic shower, where the targeted composite particle is blown apart.

A boson probe with energy on the order of $Q^2 \geq 2 \text{ GeV}^2$ can probe a hadron and interact with a single quark within this particle. As a result, information on the internal structure of a hadron, like a proton or neutron, can become available. If the constituents of a proton are point-like particles, then quantities known as structure functions of the hadronic tensors should have similar behavior to fundamental fermions.

---

12A hadron is either a baryon, a leptonic composite particle made up of three quarks, or a meson, a boson composite particle made up of quark and antiquark pairs.

13Structure functions tell us about certain physical quantities, while the hadronic tensor is a quantity made up of physical constants as well as Lorentz Invariant quantities.
Deeply Virtual Compton Scattering (DVCS): The Amplitude Equations

Let’s return to DVCS. Some physics students might remember Compton scattering, an elastic collision between a photon and an electron or another charged point particle. This interaction differs with DVCS because, as one might guess, it is not “deeply virtual”. The deep part suggests that the target charged particle is one of the constituents of the composite particle, rather than the composite particle itself; therefore, the photon penetrates the composite particle and undergoes an elastic collision with one of the constituents. The virtual part, on the other hand, refers to the fact that the photon is virtual, or not on-mass shell.

In Figure 4, just as in Figure 2, time flows from left to right. Therefore, the photon at the top left, another type of boson which interacts only with charged particles, and the incoming fermion on the bottom left interact, leading to the internal leg which is still a fermion propagator, and produce another photon and outgoing fermion.

The amplitude equation for the DVCS interaction pictured in Figure 4 is

\[ u(p) \left( -i r^\beta \epsilon_\beta \right) \frac{i (q^\gamma n_\nu + m)}{(q^2 q^\alpha - m^2)} \left( -i r^\mu \epsilon_\mu \right) u(k), \]  

where \( r \) is the charge of a quark. Again, the spin orientation is not specified in (30), but I later assigned spin orientations to the incoming and outgoing fermion spinors in (30) in Maple. For the photon’s transverse polarization, the resulting DVCS amplitude equations are compiled neatly in Table 3.

<table>
<thead>
<tr>
<th>Spin Orientation</th>
<th>Amplitude Equations for DVCS Interaction</th>
<th>Preferred or Not Preferred</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both spin up</td>
<td>[ \frac{1}{\sqrt{p_0 + m_1 \sqrt{p_0 + m_2}}} (-\epsilon_1 + i \epsilon_2)(\epsilon_1 + i \epsilon_2)i r^2 \left[ p (q_0 + m_3) k_3 + (q_0 - m_3) (p_0 + m_2) (k_0 + m_1) \right] \cos \left( \frac{\theta}{2} \right) ]</td>
<td>Preferred</td>
</tr>
<tr>
<td>Incoming spin up/Outgoing spin down (Spin Flip)</td>
<td>[ \frac{1}{\sqrt{p_0 + m_1 \sqrt{p_0 + m_2}}} (-\epsilon_1 + i \epsilon_2)(\epsilon_1 + i \epsilon_2)i r^2 \left[ p (q_0 + m_3) k_3 - (q_0 - m_3) (p_0 + m_2) (k_0 + m_1) \right] \sin \left( \frac{\theta}{2} \right) ]</td>
<td>Not Preferred</td>
</tr>
<tr>
<td>Incoming spin down/Outgoing spin up (Spin Flip)</td>
<td>[ \frac{1}{\sqrt{p_0 + m_1 \sqrt{p_0 + m_2}}} (-\epsilon_1 + i \epsilon_2)(\epsilon_1 + i \epsilon_2)(-i r^2) \left[ p (q_0 + m_3) k_3 - (q_0 - m_3) (p_0 + m_2) (k_0 + m_1) \right] \sin \left( \frac{\theta}{2} \right) ]</td>
<td>Not Preferred</td>
</tr>
<tr>
<td>Both spin down</td>
<td>[ \frac{1}{\sqrt{p_0 + m_1 \sqrt{p_0 + m_2}}} (-\epsilon_1 + i \epsilon_2)(\epsilon_1 + i \epsilon_2)i r^2 \left[ p (q_0 + m_3) k_3 + (q_0 - m_3) (p_0 + m_2) (k_0 + m_1) \right] \cos \left( \frac{\theta}{2} \right) ]</td>
<td>Preferred</td>
</tr>
</tbody>
</table>

Table 3: DVCS amplitudes and preferences.
Collapsing the Amplitude: Finding the Controlling Dirac Bilinear for DVCS

Unlike the weak interaction, the electromagnetic interaction of Figure 4 does not prefer a spin flip. Further, we see the symmetry apparent in Figure 4 is reflected in the amplitude equations of Table 3, specifically in the both spin up and both spin down orientations; in fact, these amplitudes are exactly the same! These different spin preferences recorded in Table 2 and 3 suggest that the weak interaction and electromagnetic interaction have different dominating operators.

If we collapse (30), the resulting equation (again leaving out the propagator denominator for clarity) is

\[ \Pi(p) \left( -i \epsilon_\beta \epsilon_\mu \right) \left\{ \left( g^{\beta \nu} g^{\mu \eta} - g^{\beta \mu} g^{\nu \eta} + g^{\beta \eta} g^{\nu \mu} \right) \gamma_\eta + i \epsilon^{\beta \nu \mu \eta} \gamma_5 \gamma_\eta \right\} q_\nu + \gamma^\beta \gamma^\mu m \right\} u(k). \]  

(31)

This can be further collapsed to:

\[ \Pi(p) \left( -i \epsilon_\beta \epsilon_\mu \right) \left\{ \left( g^{\beta \nu} \gamma^\mu - g^{\beta \mu} \gamma^\nu + g^{\nu \mu} \gamma^\beta + i \epsilon^{\beta \nu \mu \eta} \gamma_5 \gamma_\eta \right) q_\nu + \gamma^\beta \gamma^\mu m \right\} u(k). \]  

(32)

Following the same line of logic as we did for (29), the major controlling operator for (32) is \( \gamma^\mu \). Thus, we see from DVCS that the \( \gamma^\mu \) and \( \gamma^\mu \gamma^5 \) operators tend to preserve spin orientation, whereas for the weak interaction the operator \( \sigma^{\mu \nu} \) prefers a spin flip. With all of this in mind, it is now time to begin to address those earlier questions of why one would even try to make these comparisons between the weak interaction and DVCS; to accomplish this, we’ll need to understand how these interactions play into the parton model.
Chapter 4: The Parton Model

Partons

Recall that, for deep inelastic scattering, a boson probe with the appropriate energy can have a hard (elastic) interaction with one of the constituents of a hadron; these constituents are also known as partons, and can be one, all, or some combination of the constituents of a proton (such as quarks, gluons, etc) or of another hadron. Despite the inelastic nature of the interaction, the hard scattering between the boson and the parton can usually be separated from the messy substructure of a proton, referred to as the soft part or the soft interaction. This is the heart of the parton model, which reveals information about the internal structure of hadrons.

The Hadronic Tensor

In order to gain an appreciation for the parton model, it is necessary to return to a quantity I introduced briefly in chapter 3: the hadronic tensor. While I stated earlier that the hadronic tensor is a quantity made up of physical constants as well as Lorentz invariant quantities, it is worth digging a little deeper. First of all, why would a hadronic tensor be made up of such objects? Hadrons are objects which are extended bodies; they are composed of fundamental constituents. Therefore, it is necessary when constructing these tensors to represent them as a combination of Lorentz invariant quantities, namely the hadron and the boson’s four-momenta, as well as physical constants like mass. Further, why do we even care about such a quantity? We care about the hadronic tensor because it is one of the quantities that makes up the full amplitude equation, as well as the cross section equation, which represents the entire scattering process (with the other quantity being the leptonic tensor). This tensor is sometimes said to “parametrize our complete ignorance” about what is happening within the hadron [2].
Parton Distribution Functions (PDF’s) and Generalized Parton Distributions (GPD’s)

As stated in the first paragraph, the hard interaction can be separated from the soft interactions taking place in the proton. This manifests in the cross section equation, where something known as the parton distribution function parametrizes the soft interactions that take place in the proton, while the hard interaction between a boson and parton is represented by its own differential cross section equation. In the case of a hadronic shower, this cross section could look something like:

\[
\left( \frac{d\sigma}{dt\, ds} \right)_{LP \rightarrow LX} = \sum_i \int dx f_i(x) \left( \frac{d\sigma}{dt\, ds} \right)_{Lq_i \rightarrow Lq_i},
\]

where \( \left( \frac{d\sigma}{dt\, ds} \right)_{LP \rightarrow LX} \) is the cross section of the entire interaction, where the subscript \( LP \rightarrow LX \) represents the full interaction, a lepton interacting with a proton, resulting in a hadronic shower. Meanwhile, \( \left( \frac{d\sigma}{dt\, ds} \right)_{Lq_i \rightarrow Lq_i} \) is the cross section of the hard interaction between the boson and parton, while \( \sum_i \int dx f_i(x) \) is the parton distribution function, representing the soft interaction and summing up over all possible partons within the nucleon.

However, when we want to consider non-forward limit, these parton distribution functions are known as generalized parton distributions (GPDs). As one might expect, these two distributions depend on different things; the parton distribution functions depend on \( x \), the fraction of momentum that an individual parton carries, whereas GPDs depend on other properties in addition to this fraction of momentum between a nucleon and boson, namely \( t \) and \( \xi \), a property known as skewness.\(^{14}\)

Further, GPDs are classified by a more relevant property known as chirality, formally defined as how a Dirac object (a bilinear, spinor, etc) transforms under a left handed or right handed transformation. For us, this means we only really consider how the interactions affect a quark’s (parton’s) spin orientation. If the chirality is unaffected, which manifests as no change in spin orientation, we can extract chiral even GPDs from this interaction, whereas for a spin flip interaction, chiral odd GPDs are extracted. Thus, a clever student might guess that for the cases of the weak and DVCS interactions presented earlier, the weak interaction has chiral odd GPDs while DVCS or electromagnetic interactions have chiral even GPDs. In total, there are eight GPDs, four chiral even and four chiral odd, which can be accessed through a process known as factorization of exclusive processes.

\(^{14}\)Skewness describes the fractional change in momentum between the initial and final nucleon states, while \( t \) is the invariant momentum transfer between nucleon states (related to the t-channel).
Inclusive vs Exclusive Processes

What are exclusive processes? An exclusive process is one in which the final-state products are distinct; in layman’s terms, this process is one where we meticulously keep track of every particle in our interaction (primarily in the cross section equation). This is in contrast to inclusive processes where, as one might guess, we don’t know what all of the final-state products are, and so we often sum up over all of the possible final-states. A great example of an inclusive process in the parton model is when the probing boson excites a parton, resulting in a hadronic shower. The only thing we know in this case is that there are a lot final-state products, and that we must sum over all of these possibilities.

Hard Scattering and Quark Freedom

Before returning to the bigger picture though, it is worthwhile to question how exactly any hard scattering is achieved at the partonic level. After all, it seems odd that a boson can interact with a quark as if it were a free particle when in fact quarks are constantly interacting strongly with one another. What can allow us to treat quarks as free particles? The parton model relies on the asymptotic freedom in quantum chromodynamics (QCD); this essentially means that at short distances the effects of QCD are weak.\(^\text{15}\) In this case, quarks can then be treated as free particles and thus they are not interacting with each other in a strong way. This lack of strong interactions between quarks then allows a boson to probe or interact with a quark. What’s more, this also allows us to use only first-order (non-perturbative) Feynman diagrams to describe these interactions (see Figure 5).

The Handbag Model and Well-Known GPDs

Now that we know what exclusive processes are, we can return to the bigger picture of the factorization process and what is known as the factorized handbag model. In short, this is no different than the point I made earlier, which was that the hard interaction at the partonic level, considered the top of the handbag in the handbag model, can be factorized from the soft processes of the spectator quarks, the bottom of the handbag model. While this has only been proven for longitudinally polarized photons in meson production thanks to Frankfurt, Collins, and Strikman, we assume that factorization still holds for transversely polarized photons (or bosons) as well; from a field theory level, this is because the leading order term of the transverse case is suppressed by the same amount as the next-to-leading-order longitudinal term [4]. Because the longitudinal case is leading-order, and the transverse case is suppressed, it is assumed (without proof) that the factorization should hold for the transverse case [5].

\(^{15}\)QCD is a type of quantum field theory dealing with the strong interaction between quarks and gluons. As stated before, these are the fundamental particles which make up composite hadron particles like the proton and neutron.
Complicated theory aside, the factorization process and the parton model can perhaps be best illustrated by the following Feynman diagram.

![Feynman Diagram](image)

Figure 5: The handbag model. The dashed line represents the factorization of the hard part (the top of the diagram) from the soft part (the bottom half of the diagram). Either the weak interaction or DVCS would fit above the dashed line which separates the hard interaction from the soft interactions taking place inside of the nucleon.

We see from the bottom of the handbag model in Figure 5 how the GPDs come into the picture. To restate a point made in a previous section, GPDs “contain a wealth of information about the quark and gluon structure of the nucleon, well beyond what can be learned from the usual parton densities.” [7] In particular, GPDs yield important information about both the spin and momentum distributions of the constituent particles of hadrons. We know from earlier that there are eight GPDs in total, four chiral even \((H(x, \xi, t), E(x, \xi, t), \tilde{H}(x, \xi, t), \text{ and } \tilde{E}(x, \xi, t))\) and four chiral odd \((H_T(x, \xi, t), E_T(x, \xi, t), \tilde{H}_T(x, \xi, t), \text{ and } \tilde{E}_T(x, \xi, t))\), where this designation of even or odd is based on how a GPD affects a quark’s spin.

But what do these GPDs look like, or how do we express these GPDs? It turns out that, just as we can express the amplitudes of the soft interaction in terms of different GPDs, we can also extract equations for GPDs from different combinations of these amplitudes.\(^1\) These amplitudes involve only the target nucleon, the quark that interacts with the boson, the returning quark, and the final-state nucleon, with the propagating spectators. In both the

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\(^1\)In particular, GPDs are linear combinations of different amplitudes. See M. Diehl’s article for more.
weak interaction and DVCS, we saw how amplitudes can be collapsed into one primary Dirac bilinear which describes the most prominent or preferred behavior of a system. Thus, as one might guess, the GPDs (which are expressed as combinations of these amplitudes) have different dominating operators depending on whether the GPD is chiral even or chiral odd. Further, these soft amplitudes are typically represented as $A_{\Lambda \lambda', \Lambda' \lambda'}$ where $\Lambda$ and $\Lambda'$ represent the spin of the initial and final nucleons, while $\lambda$ and $\lambda'$ represent the spin of the outgoing and incoming quark. For completeness, the amplitudes of the hard interaction (quark and boson go to quark and meson) are represented as $g_{\Lambda \gamma, 0 \lambda'}$, where $\Lambda \gamma$ is the boson’s polarization. To express the entire interaction pictured in the factorized handbag model, from the soft and hard amplitude equations we can glue the hard part and soft part together to form

$$f_{\Lambda \gamma, 0 \Lambda'} = \int dX \sum g_{\Lambda \gamma, 0 \lambda'} \otimes A_{\Lambda \lambda, \Lambda' \lambda'},$$

where we are summing over $\lambda$ and $\lambda'$. This equation can then be used to calculate the cross section equations [5].

Instead of looking at cross sections though, let’s turn our attention to the well-known GPDs contained in Table 4 (see next page). For the GPDs presented in Table 4, the most recent experimental data yields values for the helicity distribution and the tensor charge. Current values of the helicity distribution $g_1(x)$ are, for an up quark, $\Delta q^f = \Delta u = 0.82 \pm 0.07$, while, for a down quark, $\Delta q^f = \Delta d = -0.45 \pm 0.07$, making $\Delta u - \Delta d = 1.27 \pm 0.14$ [6]. Meanwhile, current values of the tensor charge $\delta q^f$ are, for an up quark, $\delta q^f = \delta u = 0.59^{+0.14}_{-0.13}$, and, for a down quark, $\delta q^f = \delta d = -0.20^{+0.05}_{-0.07}$, making $\delta u - \delta d = 0.79^{+0.19}_{-0.20}$ [6]. We use this data to reverse engineer our models for the GPDs, which is at the heart of phenomenology.
Coming Full Circle (Conclusion)

GPDs like the ones which in appear in Table 4 are extracted from the amplitudes of the weak and electromagnetic interactions discussed in this thesis. In the electromagnetic interaction we saw that it did not prefer a spin flip, therefore the dominating operators, $\gamma^\mu$ and $\gamma^\mu \gamma^5$, do not exhibit spin flip behavior. From these amplitudes, we extract chiral even GPDs. Therefore, as one might expect, the dominant operator structure of the chiral even GPDs are $\gamma^\mu$ and $\gamma^\mu \gamma^5$ [7]. However, in the weak interaction we saw that they did prefer a spin flip, and therefore the dominating operator $\sigma^{\mu \nu}$ and $\sigma^{\mu \nu} \gamma^5$ do exhibit spin flip behavior. Further, from these weak amplitudes we extract chiral odd GPDs. Therefore, the dominant operator structure of chiral odd GPDs are $\sigma^{\mu \nu}$ and $\sigma^{\mu \nu} \gamma^5$ [7].

Historically, the GPDs of the electromagnetic interaction have been much more explored than those of the weak interaction; this is due in large part to the rarity of the weak interaction as well as the fact that it takes more energy to generate. However, the weak interaction tells us more information through flavor changing. The weak boson is more

<table>
<thead>
<tr>
<th>Function</th>
<th>Name/Details</th>
<th>What it Does</th>
<th>Relationship to Measurable Quantities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1(x) = H(x,0,0)$</td>
<td>$f_1(x)$ is known as the unpolarized distribution because quarks are measured independent of helicity in this function.</td>
<td>$f_1(x)$ is defined as the probability of finding a quark or parton with momentum fraction $x$ within an unpolarized parent nucleon.</td>
<td>$\int dx (f_1^u(x) - f_1^d(x)) = q_v$ if we sum this up over all momentum fractions, this should then give us the total number of valence quarks $q_v$ (as opposed to sea quarks) of a specific flavor. Thus, integrating this over $x$ will be 2 for up quarks and 1 for down quarks.</td>
</tr>
<tr>
<td>$g_1(x) = H_T(x,0,0)$</td>
<td>$g_1(x)$ is known as the helicity distribution.</td>
<td>$g_1(x)$ is defined as the probability of finding a parton with momentum fraction $x$ and spin polarized parallel to the longitudinally polarized parent nucleon’s spin minus the probability of finding another parton with the same momentum fraction but whose spin is polarized anti-parallel to the parent nucleon’s spin, or: $q^+ - q^-$</td>
<td>$\int dx (g_1^u(x) - g_1^d(x)) = \Delta q_T^f$ $\Delta q_T^f$ is the axial charge of the nucleon for a quark of flavor $f$.</td>
</tr>
<tr>
<td>$h_1(x) = H_T(x,0,0)$</td>
<td>$h_1(x)$ is known as the transversity distribution.</td>
<td>$h_1(x)$ is defined as the probability of finding a parton with momentum fraction $x$ and transverse spin polarized in the direction of the transversely polarized parent nucleon’s spin minus the probability of finding a parton with the same momentum fraction $x$ and spin in the opposite direction, or: $q^T - q^\perp$</td>
<td>$\int dx (h_1^u(x) - h_1^d(x)) = \delta q_T^f$ $\delta q_T^f$ is known as the tensor charge for a quark of flavor $f$.</td>
</tr>
</tbody>
</table>

Table 4: Well-known GPDs and their properties.
likely than the photon to be able to probe a strange or charm sea quark within the nucleon that is bound to its antiquark pair. The reason behind this is that the weak boson only sees weak isospin, whereas the photon only sees charge. If a photon is interacting with the constituents of a nucleon, it is going to be more likely to interact with a valence up quark rather than a bound sea quark. So, a weak boson probe shows us a wider variety of interactions by changing the flavor of the target quark. In addition to flavor changing, the weak interaction gives us access to the tensor charge, which lets us assess the accuracy of chiral odd models which were previously inaccessible.
Bibliography


