Radiative Cooling of Silicon Mirror for Gravity Wave Detection

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Radiative Cooling of Silicon Mirror for Gravity Wave Detection

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Abstract

Gravitational waves are distortions in the fabric of space-time caused by the acceleration of masses, such as rotating black hole binaries. The Laser Interferometer Gravitational wave Observatory (LIGO) is attempting to detect cosmic gravitational waves. My experiment involves investigating the feasibility of cooling mirrors to reduce thermal noise and hence increase the sensitivity of the detector for the Third Generation LIGO. I am investigating radiative cooling of silicon mirror substrates to 120K using high-emissivity coatings. I will measure the emissivity and the mechanical Quality factor (Q) of the wafer with and without using high emissivity coating.
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Chapter 1

Gravitational Waves

Gravity is one of the four fundamental forces of the universe, and is central to the quest for the Theory of Everything. Until now, our sense of the Universe has been predominantly provided by the wide spectrum of electromagnetic waves. For instance, optical astronomy gave us the Hubble’s law of expansion of universe as well as providing us a peek at the universe through the Hubble Deep Field. We learned about cosmic background radiation through radio astronomy, and found evidence of a massive black hole in the center of Milky Way through infrared astronomy. Gravitational waves, the messenger of gravity, offers us a new sense with which to probe the universe. This chapter provides a brief introduction to gravitational physics or gravitational wave astronomy.

In 1916, Albert Einstein published the Theory of General Relativity where he predicted the existence of gravitational waves. According to the theory concentrations of large amount of mass warp space-time. Changes in the distribution of mass cause a distortion in the space-time fabric that propagates through the universe at the speed of light. This propagating disturbance of space-time is called gravitational wave.

Figure 1.1: Illustration of curvature of space-time due to the presence of mass. Space and time is being dragged around a massive body, the Earth. The arrows show the path to be taken by an object placed at that point, e.g. the orbit of the Moon around the Earth.

The concept of gravitational wave came after Einstein finished his work on the Special Theory of Relativity, and wanted to incorporate gravity into the picture. Einstein noted that according to the notion of causality in relativistic physics, no signal can travel faster than the speed of light. But the instantaneous action-at-a-distance of gravity under the Newtonian gravitation theory seemed to violate this prohibition. To amend for
this, Einstein proposed in his Theory of General Relativity the existence of gravitational waves which carry gravitational signal at the same speed as the electromagnetic waves [7]. Hence, causality is not violated!

The gravitational field of a massive body is essentially the way in which it curves space-time. Given this curvature, bodies travel through space-time along paths that are as straight as possible. The "force of gravity" isn’t a force separate from the space-time arena but rather the force needed to keep an object moving along the geodesics - the shortest distance between two points in the curved space-time [6]. The path of the Moon around the Earth is determined by the curvature of the space-time generated by the two masses, as shown in Fig.1.1.

Even though it has been almost a century since Einstein predicted the existence of gravitational waves, they are yet to be detected because of their extremely weak nature. Bigger mass produces bigger gravitational waves, and those produced by massive astrophysical objects can be detected at the Earth. Let us visualize the size of distortion produced in Earth by gravitational waves from two black holes revolving around each other. The change in length of a meter scale produced by this gravitational wave is $10^{-21}$ m [2]. This is less than a trillionth of the size of an atom - an extraordinarily small change in length. In fact, Einstein himself had no confidence that gravitational waves would ever be detected [2]!

Nevertheless, Joseph Weber began the saga of detecting gravitational waves experimentally around 1960. With piezoelectric-electric chips, he monitored the minute vibrations of massive aluminum cylinders, hoping that they will be set to oscillation by passing gravitational waves. And they did - according to Weber [9]. In 1969, he announced detecting coincident signals from his pair of detectors, likely of cosmological origins. A community of gravitational wave experimenters were born to verify the discovery by Weber. Unfortunately, no other experiment was able to verify Weber’s observation, but the saga of gravitational wave detection continues.
Over the years, Weber’s resonant mass detectors have been replaced by smarter and more sensitive detectors, and our greatest hope now lies on the large interferometer based detectors. There are two such endeavors: the United States based Laser Interferometer Gravitational wave Observatory (LIGO) project, and the Italian-French based VIRGO project. Continuous improvements are being made to these detectors and there is optimism that detection is near. My thesis project is involved with investigating the feasibility of one such improvements for the Third Generation LIGO detector. But before I get to that, I’ll continue with an overview of gravitational waves, and the practice of gravitational wave detection.

1.1 The Nature of Gravitational Waves

Newton’s theory of gravitation predicts that the gravitational force produced by a mass at a distance \( r \) away from its present position, always has the \( 1/r^2 \) form, irregardless of the motion of the mass. General relativity fixes the problem posed by accelerating sources of gravitational field. It proposes that the change in gravitational field, i.e. the gravitational waves, propagate at the speed of light, analogous to electromagnetic waves. Properties of gravitational waves can be explained by drawing analogy with electromagnetic waves:

1. Electromagnetic waves (EMW) are oscillating electromagnetic fields propagating through space-time, while gravitational waves (GW) are propagation of distortions of space-time itself due to accelerating mass as in Fig.1.3.

![Figure 1.3: Artist’s rendition of a binary-star merger creating gravitational waves. Changes in the gravitational field produced by the motion of the masses are being propagated by gravitational waves.](image)

2. Both EMW and GW travel at speed \( c \), and carry energy and momentum with them.

3. EMW are strongly interacting and experience strong absorption and scattering in interaction with matter. But GW are very weakly interacting and have essentially no absorption or scattering with matters through which they are propagating - a reason why they are extraordinarily difficult to detect.

4. EM waves are typically produced by acceleration of charges \textit{within} an object, and often have wavelengths much smaller than the object. GW are produced by bulk motion of the object \textit{itself},
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(a) Lines of force for electromagnetic waves propagating along the x axis.

(b) Lines of force for plane gravitational waves propagating along the z axis [2].

Figure 1.4: Lines of forces for electromagnetic and gravitational waves.

...and have wavelengths much larger than the size of the object. The frequency of EMW from astronomical sources ranges from \(10^3\) Hz (wavelength \(10^5\) m) for radio waves to \(10^{23}\) Hz (\(10^{-15}\) m) for gamma rays. For gravitational waves, the range is from \(10^{-16}\) Hz (\(10^{24}\) m) to \(10^2\) Hz (\(10^6\) m).

5. The two components of electromagnetic waves, the electric field and the magnetic field are at \(90^\circ\) to each other, and the direction of propagation is at right angle to these fields. For gravitational waves, the direction of propagation is also at right angle to the field, but the two polarizations are at \(45^\circ\) to each other as shown in Fig. 1.4b.

6. The dominant mode of electromagnetic radiation is dipolar. The dominant mode of gravitational radiation is quadrupolar. It has a quadratic dependence on the positions of the generating charges, and causes a relative shearing" of the positions of receiving charges (Fig. 1.4).

1.2 Gravitational Waves in General Relativity

In Special Theory of Relativity, the space-time interval \(ds\) between any two neighboring points is given by

\[
d s^2 = c^2 dt^2 + dx^2 + dy^2 + dz^2
\]

(1.1)

\[
d s^2 = \eta_{\mu\nu} dx^\mu dx^\nu,
\]

(1.2)

where \(\eta_{\mu\nu}\) is the Minkowski metric of a "flat" space-time,

\[
\eta_{\mu\nu} = \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

(1.3)

In the Theory of General of Relativity, the space-time is no longer "flat", but is curved in order to represent gravitation. The general statement of the definition of the space-time interval is

\[
d s^2 = g_{\mu\nu} dx^\mu dx^\nu,
\]

(1.4)
where the metric tensor $g_{\mu\nu}$ contains all the information about the space-time curvature. However, we are interested only in the special case of a small perturbation to flat space-time. The metric $g_{\mu\nu}$ can be written in the form

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},$$

(1.5)

Here $h_{\mu\nu}$ represents the metric perturbation away from the Minkowski space. In the weak-field limit, the non-linear Einstein equations can be approximated as linear equations. The transverse traceless gauge (TT gauge) is used for the construction of $h_{\mu\nu}$. In this gauge, the coordinates are marked out by the world lines of freely-falling test masses. The weak-field limit of Einstein’s field equation becomes a wave equation

$$(\nabla^2 - \frac{1}{c^2} \frac{\delta^2}{\delta t^2})h_{\mu\nu} = 0,$$

(1.6)

The elements of $h_{\mu\nu}$ take the form $h(2\pi f t - k \cdot x)$, with $f = |k|/2\pi c$. This represents a plane wave propagating in the direction $\hat{k} \equiv k/|k|$ with speed $c$.

For a wave propagating along the $\hat{z}$ axis, the TT gauge $h_{\mu\nu}$ has the form

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & 0 \\ 0 & b & -a & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(1.7)

which can be written as the sum of two components, $h = a\hat{h}_+ + b\hat{h}_\times$. These basis tensors "h-plus" $\hat{h}_+$ and "h-cross $\hat{h}_\times$ represents the two orthogonal polarizations for waves polarizing along the $\hat{z}$ axis. The tensors are represented by

$$\hat{h}_+ = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(1.8)

$$\hat{h}_\times = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(1.9)

The two basis vectors have their principal axis rotated by $45^\circ$ from one another. A perturbation by one of these basis tensors, momentarily lengthens distance along one direction, and shortens it in an orthogonal direction.

In conclusion, gravitation is not a force, but a phenomenon of geodesic motion through curved space-time. From a relativistic point-of-view, a freely-falling mass, i.e. an object not subject to influences of non-gravitational force, does not accelerate when a gravitational wave passes. Hence, a set of freely-falling masses can be used to define a coordinate system in space-time. It is the space between the
freely-falling test masses which gets curved when a gravitational wave pass through them, but their coordinates remain unchanged.

1.3 Evidence of Gravitational waves.

Gravitational wave has not been observed directly yet, but strong indirect evidence for its existence was found from the observations of Russell Hulse and Joseph Taylor. They studied the neutron star binary pulsar system PSR 1913+16 which they have discovered in 1974 and determined it to be a binary system with a revolution period of 8 hours. The pulsar in this binary system has a rotational period of 17 seconds, which they used to determine the masses, separation and ellipticity of orbit of the two neutron stars. Hulse and Taylor monitored these pulsar signals over many years and observed a speedup of the orbital [10]. They demonstrated that the motion of the pulsar around its companion could not be under-
stood unless the dissipative reaction force associated with gravitational wave production were included. They found that the system radiates away energy, presumably in the form of gravitational waves and the two neutron stars are slowly spiraling in towards one another resulting in a speeding up of the orbital period. The value of the speedup they recorded is in complete agreement with the predictions from general relativity (Fig. 1.6). In 1993 they received the Nobel Prize in Physics for their work.

1.4 Gravitational Wave Spectrum

Gravitational waves offers us a new tool with which to probe the universe. This provides the motivation for performing gravitational wave astrophysical observation by studying the wide spectrum of gravitational waves from different astrophysical sources. Figure 1.7 shows the various astrophysical sources producing gravitational waves with their possible wavelengths, frequencies and strain. Strain is the fractional change in length produced by the wave and is defined as,

\[ h \equiv \frac{\Delta L}{L} \] (1.10)
1.4.1 Sources of Gravitational Waves

According to General Relativity, gravitational waves are produced by massive accelerating objects and a variety processes in the Universe may give rise to detectable gravitational waves here on Earth. Here is a description of a few mentioned in Fig. 1.7.

1. Relic Background: This is the stochastic (random) signal from the Big Bang, consisting of quantum fluctuations in the initial explosion that have been amplified by the early expansion of the Universe.

2. Binary Background: This is another stochastic signal arising from thousands of rotating binary systems emitting gravitational waves continuously in overlapping frequency bands. The individual signals are unresolvable. At longer wavelengths are pairs of supermassive black holes while at shorter wavelengths are binary white dwarf stars within our own Galaxy.

3. Super-Massive Black Hole Binaries (SMBHB): Occasionally some supermassive black hole system will merge, producing a huge burst of gravitational waves at milliHertz frequencies.

4. White Dwarf Binaries (WDB): Above the white dwarf stochastic background are a few thousand individually-resolvable white dwarf binary systems in our Galaxy. Many of these will be known calibrators for future gravitational wave detectors, since they have already been charted with conventional astronomy.

5. Extreme-Mass Ratio Inspirals (EMRI): These are compact stellar remnants consisting of white dwarfs, neutron stars, or stellar-mass black holes, in the process of being captured and swallowed by a supermassive black hole.

6. Black Hole Binaries (BHB): These are rotating binary systems consisting of two stellar-mass black holes.

7. Neutron Star Binaries (NSB): These are rotating binary systems consisting of two neutron stars, rotating around each other.

8. Neutron stars (NS): These individual neutron stars which are spinning non-axisymmetrically, e.g. pulsars.

1.4.2 Astrophysical Interest for Detecting Gravitational Waves

The direct observation of gravitational waves is central to the understanding of frontier astrophysical phenomena such as black holes, pulsars, quasars, the final destiny of stars, the big bang, etc. There is a need for experimental confirmation of the existence of gravitational waves and for exploration of GW sources. The observations carried out by the gravitational wave detectors will provide fundamental and new information by:

- direct measurement of the polarization states of GW
- direct measurement of the speed of GW
• direct measurement of strong field gravity through the observation of the gravitational waves from black holes.

• direct observation of compact stellar systems such as neutron star/neutron star, black hole/black hole and black hole/neutron star binaries will provide detailed information of the relativistic equations of motion.

1.4.3 Detectors of Gravitational Waves

Gravitational wave detectors are devices designed to detect extraordinarily small distortions in space-time within a particular frequency range. Because of the wide spectrum of gravitational waves, there are several different kinds of detectors for various frequency ranges. These detectors are all in different stages of development [5].

1. Cosmic Microwave Background (CMB): Several thousand years after the Big Bang, when the hot plasma of protons and electrons cooled and combined to form the first atoms, electromagnetic radiation was released into the newly-transparent Universe. In the present Universe, this cooled and red-shifted radiation can be seen as microwave background. Small fluctuations can be seen in the observed temperature across the sky, which are due to density variations in the plasma, and also due to perturbations by long-wavelength gravitational waves. The fluctuations due to GW are however difficult to separate out with the current sensitivity of the CMB detectors, but is possible with the evolution of CMB detectors.

2. Pulsar Timing and Pulsar Timing Array: Pulsars are spinning neutron stars that emit beams of electromagnetic radiation, seen as “pulses” when they sweep over the Earth. Gravitational wave alters the path length between the pulsar and the Earth, changing the pulse arrival times in fluctuating manner. A gravitational wave could be detected if two or more pulsars show a correlated pattern of fluctuations in pulse arrival times.

3. Laser Interferometer Space Antenna (LISA): This is a space mission, which has lost its funding, and hence may never fly. Three spacecraft in Solar orbit 5 million kilometers apart will use laser ranging to monitor their relative separations, and will be sensitive to changes caused by passing gravitational waves.

4. Laser Interferometer Gravitational wave Detector: An example of such ground-based detector, is the Laser Interferometer Gravitational wave Observatory (LIGO) and Ch. provides a thorough treatment of its principle of operation. My thesis project is to investigate the feasibility of an improvement to the Third Generation LIGO.
Chapter 2

Laser Interferometer Gravitational wave Observatory (LIGO)

The Laser Interferometer Gravitational wave Observatory (LIGO) is an astronomical instrument designed to probe the Universe by detecting gravitational waves. It is a National Science Foundation sponsored project being jointly conducted by California Institute of Technology and Massachusetts Institute of Technology. The LIGO detector consists of two remotely located interferometers, one at i) the DOE Hanford Nuclear Facility in the Washington State and the other at ii) Livingston Parish, Louisiana, separated by 3030 km (cite coyone). These are Michelson interferometers with resonant Fabry-Perot cavities in the 4 km long arms. The interferometer arms at the two sites are oriented for maximum coincidence sensitivity for a single gravitational wave polarization, so as to separate out real events from the background. The two sites have been chosen to be sufficiently separated so that environmental perturbations to the interferometers are expected to be independent and hence uncorrelated. The initial LIGO detectors were designed to be sensitive to GW in the frequency band 40-7000 Hz [5]. The sources of GW that LIGO might detect include binary neutron stars, black hole binaries, stellar core collapse which triggers a Type II supernova, rapidly rotating non-axisymmetric neutron stars, and possibly some relic background [4].

(a) Hanford, WA. (b) Livingston, LA.

Figure 2.1: Sites of Laser Interferometer Gravitational wave Observatory
2.1 Astrophysical Interest of LIGO

Detection of gravitational waves by LIGO will provide a new view of the universe with a high probability of uncovering phenomena not observed by electromagnetic astronomy. The astrophysical information derived from LIGO observations will include:

- the spatial and mass distribution of neutron star binary systems.
- the spatial and mass distribution of black hole binary systems.
- the internal dynamics of asymmetric supernova explosions.
- limits to or measurements of the gravitational multipole moments of pulsars.
- limits or observations of the gravitational wave background from the earliest epoch of cosmic evolution.

2.2 Michelson Interferometer

Each LIGO interferometer is fundamentally a Michelson interferometer designed to detect differential motion between its arms as small as $10^{-18}$ m rms. The Michelson interferometer originated from an experiment Albert Michelson and Edward Morley performed in 1887, in an attempt to detect the apparent shift in the speed of light due to the Earth’s motion through the ether. The device consists of a source of light, a beam-splitter which is a partially reflecting mirror, and two mirrors located some distance away from the beam-splitter in two orthogonal directions. Light transmitted through the beam-splitter travels towards one mirror, while the reflected mirror travels down the orthogonal arm. The lights beam reflected from the end mirrors travel back and superpose at the beam-splitter (Fig. 2.2).

Michelson Interferometer compares the amount of time the light beams in the two arms spend, to compare their paths. At each front-surface reflection, the light will undergo a phase inversion. For the superposed light at the beam-splitter, if the paths of the two lights differ by a zero or whole number of wavelengths, there will be destructive interference giving a weak or no signal at the detector. If they differ by a whole number and a half wavelengths there will be constructive interference and a strong signal will be detected. The power of the light detected at the photo-detector will be

$$P_{\text{out}} = P_{\text{in}} \sin^2(\Delta \phi)$$

(2.1)
CHAPTER 2. LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY (LIGO) 17

where, $P_{in}$ is the input power of the laser and $\Delta \phi$ is the phase difference.

### 2.3 Principles of Operation of LIGO interferometers

When a gravitational wave passes through the interferometer, the space-time in the local area is altered resulting in an effective change in length of the arms. The length of each arms of the LIGO interferometers is 4 km under resonant conditions, i.e. when there is no GW passing. There are two Fabry-Perot cavities on the two arms which result in multiple-passes of the beam. Due to incoming GW the distances along the arms of the interferometer are shortened and lengthened, causing the beams to become slightly out of phase with the incoming light (Fig. 2.3). The cavity will therefore periodically get very slightly out of resonance and the beams which are tuned to destructively interfere at the beam-splitter, will have a very slight periodically varying detuning. After several trips down the Fabry-Perot cavities the two separate beams leave the arms and recombine at the beam splitter. The amplified detuning will result in a measurable signal at the photo-detector.

### 2.4 Limits to Performance

In actual operation, sources other than gravitational waves can cause movement in the optics producing similar effects to real gravitational wave signals. Detection is limited by a number of noise sources which are described below. The strain or the fractional change in length of targeted waves of LIGO is $10^{-21}$ m, which results in an arm length change of only $10^{-18}$ m (due to the use of 4 km long baseline), a thousand times smaller than the diameter of a proton. The challenge of LIGO is to make the instrument sufficiently sensitive to detect this extraordinarily small change. LIGO is utilizing special interferometry techniques, state-of-the-art optics, highly stable lasers, and multiple layers of vibration isolation to increase its sensitivity and minimize the background noise [?].

- **Seismic Noise**: This is the noise due to the vibration of the terrestrial environment. Very little can be done to control the level of this noise, but precautions can be taken against the noise leaking
into the system. Cascaded, multi-stage seismic isolation systems and pendulum suspension for test masses isolates the test masses from these seismic vibrations.

- **Optical Readout Noise:**
  - **Photon Shot Noise:** The uncertainty in the phase of a laser beam due to quantization of light into wave packets is photon called shot noise. This noise follows Poisson statistics and hence increasing the number of photons coming to the detector is a way of mitigating shot noise. Use of a resonant Fabry-Perot cavities with multiple-pass, power recycling, increased laser input power and utilization of squeezed-state light, are the few techniques undertaken by LIGO to reduce shot noise.
  
  - **Radiation Pressure noise:** Light reflecting from the mirrors impart force on the mirror due to its momentum. Fluctuations in the amplitude of the laser thus cause random forces on the optics. The forces due to intensity fluctuations are symmetric between the two arms so that they cancel out. However, for unintentional mechanical and optical asymmetries some do not cancel and this noise increases with input power. This sets an upper limit to the input
power and calls for very stable lasers.

- **Residual Gas:** Fluctuations in the forward scattering of laser light by residual gas particles is a noise source. This can be minimized by operating at lower pressure and the operating pressure of LIGO is $10^{-6}$ Torr.

- **Thermal Noise:** This is fundamental source of noise due to Brownian motion of particles on the surface of mirrors, and is explained in more details in the next chapter.
Chapter 3

Thermal Noise

Thermal noise is one of the fundamental limits to the precision of mechanical measurements, and originates from the Brownian motion of particles. It limits the degree to which the test masses of a gravitational wave detector can remain at rest. Any mechanical system which is in equilibrium with a thermal reservoir, will have mechanical resonant modes, each with energy $k_B T$, where $k_B$ is the Boltzmann’s constant and $T$ is the temperature. The Brownian motion of the particles within the system and the reservoir impart randomly fluctuating thermal forces on the mechanical system, resulting in random displacements. This thermally induced displacement noise is known as the thermal noise.

3.1 Brownian Motion

Brownian motion, the classic form of thermal noise, was first observed by the microscopist Robert Brown around 1828. He hypothesized the ceaseless jiggling motion of the small dust particles to have resulted from the action of a universal "vital force". This vital force was later explained by Einstein as arising from fluctuations in the rate of impacts of individual water molecules on a dust particle. Einstein realized that the dissipation of kinetic energy of the dust particle moving through water is dependent on this rate of impact. He related the two together to show that the mean-square displacement of a particle is:

$$\langle x^2_{\text{thermal}} \rangle \propto \frac{T \eta}{a}$$  \hspace{1cm} (3.1)
where, \( a \) is the radius of the particle and \( \eta \) is the viscosity of the fluid. This gives a link between the fluctuation phenomenon, i.e. the random displacement of the particle, and a mechanism for dissipation, the viscosity of the fluid. This relation is a basis for the fluctuation dissipation theorem, explained next.

### 3.2 Fluctuation-Dissipation Theorem

The fluctuation-dissipation theorem, originally formulated by Harry Nyquist in 1928 and later proven by H. B. Callen and his co-workers in 1951, is a tool in statistical physics for predicting the behavior of non-equilibrium thermodynamical systems. The theorem assumes that the response of a system in thermodynamic equilibrium to a small applied force is the same as its response to a spontaneous fluctuation.

For a mechanical system in equilibrium with a thermal reservoir, the fluctuation-dissipation theorem describes how the randomly fluctuating thermal forces from the reservoir seeps into the mechanical system through its mechanical impedance and results in random displacement in the system. Here mechanical impedance is the ratio of the force to the velocity induced. By using the FDT one does not have to make a detailed microscopic model of any dissipation phenomenon in order to predict the fluctuation associated with it. A macroscopic mechanical model specifying the impedance as a function of frequency can be used instead. The theorem also shows that a way to reduce thermal noise is to reduce the amount of dissipation in the system \cite{footnote7}.

#### 3.2.1 Derivation of Fluctuation Dissipation Theorem

A system is said to be dissipative if it is capable of absorbing energy when subjected to a time-periodic perturbation. The system is linear if the power dissipation is quadratic in the magnitude of the perturbation. For a linear dissipative system, the impedance can be shown to be the proportionality constant between the power and the square of the perturbation amplitude.

For example, the external force necessary to move a system with sinusoidal velocity of amplitude \( v(\omega) \) can be written as,

\[
F_{\text{ext}} = Zv
\]

or, equivalently as,

\[
v = YF_{\text{ext}}
\]

Here, the function \( Z(\omega) \) is called the impedance and the function \( Y(\omega) \) is called the admittance, where \( Y(\omega) = 1/Z(\omega) \).

The fluctuation-dissipation theorem states that the power spectrum \( F_{\text{th}}^2(\omega) \) of the fluctuating force on a system is given by,

\[
F_{\text{th}}^2(\omega) = 4k_B T \Re(Z(\omega))
\]
where $\Re(Z)$ indicates the real, i.e. the dissipative part of the impedance. The power spectrum of the system’s fluctuating motion is given as

$$x_{th}^2(\omega) = \frac{4k_BT}{\omega^2} \Re(Y(\omega))$$

(3.5)

This is saying that if the variable $x$ represents the position of some part of a mechanical system, with $x = 0$ as equilibrium, then the mean-squared fluctuation in $x$ caused by finite temperature is given by the above function of angular frequency $\omega$. The real part of the admittance describes the loss in the system. Hence to reduce displacement we need to lower either the temperature or the loss in the system.

### 3.3 Sources of Dissipation

The losses of a system can be illustrated using a damped harmonic oscillator as an example. Internal damping in any material obeys Hooke’s Law, approximated by a complex spring constant, which is a collection of both the restoring force and the damping term.

$$F = -k[1 + i\phi(\omega)]x$$

(3.6)

This means if the force $F$ is sinusoidal, the response $x$ of the spring will lag the force by the angle $\phi(\omega)$. That is, the physical material will continue to stretch after the external stress is removed. This phenomenon is called anelasticity, and the function $\phi(\omega)$, known as the loss angle represents the degree of anelasticity of the spring. A fraction $2\pi\phi$ of the energy stored in the oscillatory motion is being dissipated during each cycle. The complex spring constant is associated with damping, and the fluctuation-dissipation theorem guarantees that the damping generates mechanical noise.

The equation of motion of the oscillator is

$$F_{ext} = m\ddot{x} + k(1 + i\phi)(x - x_f)$$

(3.7)

From Eq. 3.2, the mechanical impedance at the mass is $Z \equiv F_{ext}/v$. Replacing $x = v/i\omega$ and $\ddot{x} = i\omega v$ into Eq. 3.7,

$$Z(\omega) = i\omega m + \frac{k}{i\omega} + \frac{k\phi}{\omega}$$

(3.8)

The admittance is the inverse of the impedance,

$$Y(\omega) = \frac{\omega k\phi(\omega - m\omega^2)}{(k - m\omega^2)^2 + k^2\phi^2}$$

(3.9)

According to the fluctuation-dissipation theorem, the mean-squared thermal noise in a simple harmonic oscillator is $4k_BT/\omega^2$ times the real part of the admittance, or

$$x_{th}^2(\omega) = \frac{4k_BTk\phi(\omega)}{\omega[(k - m\omega^2)^2 + k^2\phi^2]}$$

(3.10)
For our familiar viscous damping, with a frictional force proportional to velocity \( F_{\text{fric}} = -\gamma v \), the loss angle is proportional to frequency.

\[
\phi(\omega)_{\text{viscous}} = \frac{\gamma}{k} \omega
\]  \hspace{1cm} (3.11)

However, there are other sources of damping which give rise to loss angle with different frequency dependence. One such source is the loss due to internal friction or structural damping, which is frequency-independent.

\[
\phi(\omega)_{\text{structural}} = \text{constant}
\]  \hspace{1cm} (3.12)

A third source of damping is the frequency-dependent thermoelastic damping, and I’ll explain this in greater detail. However, first a brief description of the quality factor \( Q \), and its relation with thermal noise.

### 3.3.1 Quality Factor, \( Q \)

![Figure 3.2: The Quality Factor \( Q \) of an oscillator, measures in frequency and time domain [7].](image)

The quality factor \( Q \), of an oscillator is a dimensionless measure of how small the dissipation is at the resonant frequency. It is the ratio of the elastic restoring force to dissipative force and is defined as,

\[
Q \equiv \frac{f_0}{f_{\text{fwhm}}} = \frac{\omega_0}{\omega_{\text{fwhm}}}
\]  \hspace{1cm} (3.13)
where, \( f_0 \) and \( \omega_0 \) are the resonant frequency and resonant angular frequency respectively, and \( f_{\text{fwhm}} \) and \( \omega_{\text{fwhm}} \) are the Full Width Half Maximums of the oscillator. Since, the Q gives the ratio of the energy stored in the system to the energy lost per cycle, a system with high-Q is a low-loss system and vice-versa.

Nyquist observed, that if the frictional losses of a torsional pendulum is made small the resonant frequency becomes more sharply peaked - the thermal noise gets concentrated around the resonant frequency and the off-resonance thermal noise is reduced. This can be explained in terms of the equipartition theorem, which states that each quadratic term in the energy has a mean value of \( \frac{1}{2} k_B T \). For our simple harmonic oscillator,

\[
\frac{1}{2} k_B T = \frac{1}{2} k_B T
\]  \hspace{1cm} (3.14)

Therefore, the integral over all frequencies of Eq. \[3.10\] gives the mean-squared displacement of \( k_B T \), which is the total area under the curve. Therefore the higher the Q of a mechanical system, i.e. the sharper the peak at the resonant frequency, the more thermal noise is at the resonant frequency, and less is at the off-resonant frequencies. The relation between the quality factor \( Q \) and the loss angle \( \phi \) is given by,

\[
Q = \frac{1}{\phi(\omega_0)}
\]  \hspace{1cm} (3.15)

where, \( \phi(\omega_0) \) is the loss-angle at the resonant frequency. So a mechanical system with high-Q is desirable for having low thermal noise, and vice-versa.
### 3.3.2 Thermoelastic Damping

Thermoelastic damping, an example of anelasticity, is an important source of losses for thin samples which can undergo flexure. The temperature of an object is an averaged value. In the microscopic level, temperature exhibits local fluctuations, even when a object is in thermodynamic equilibrium. These fundamental fluctuations cause the material to expand or contract, depending on the sign of the fluctuation, through the coefficient of thermal expansion. The loss angle for thermoelastic damping is frequency-dependent, and is given by

\[
\tan(\delta) = \frac{\omega \tau}{2}
\]

The thermoelastic damping is a fundamental source of noise, and will be present irregardless of whether the system has a high or low mechanical Q. Since the size of the flexure of the material due to local fluctuations is dependent on both the dependent on both the temperature and on the coefficient of thermal expansion of the material, it is desirable to operate at lower temperature and use material with low expansion coefficient.

### 3.4 Silicon as Mirror Substrate

The amplitude of test mass deformations due to temperature changes is directly proportional to the substrate thermal expansion, and depends on the magnitude of the coefficient of linear thermal expansion [5]. Silicon has a special feature of null coefficient of thermal expansion for temperatures of 20 K and 120 K as shown in Fig.3.4. In theory this means that at 20 K and 120 K silicon has null thermal noise contribution from thermoelastic damping. This makes it an excellent candidate for test mass substrate for cryogenic gravity wave detector.

(a) Linear Expansivity is 0 at 18 K and 120 K for Silicon. The slope is 0 at 120 K.

(b) Linear Expansivity is the derivative of strain.

Figure 3.4: Coefficient of Linear Expansion of Silicon

### 3.5 Cryogenic Detector

For interferometers operated at room temperature mirror thermal noise is a dominant noise source in the hundred hertz region in which LIGO is interested [2.4]. Technological improvement in other
areas, such as the use of squeezed light (reference) to reduce shot noise in the range of interest is being overshadowed by the dominance of thermal noise in the same region. As discussed in Ch. 3 a promising way to significantly decrease the magnitude of thermal noise is to cool the detector. Lowering the sensor temperature have successfully extended range of several astronomical detectors, such as CCD camera and radio receivers, and the same technique can be applied to future gravitational wave detectors. In addition to reducing local fluctuations by lowering the temperature of the detector, the magnitude of thermoelastic damping can be dramatically reduced by using silicon as the test mass and by operating at 120 K. The Third Generation LIGO is hence proposed to be operated at cryogenic temperature, and silicon is one of the candidates for the mirror substrate as well as for the suspension fibers of the mirror.
Chapter 4

Radiative Cooling of Silicon Wafer

The design for the Third Generation LIGO replaces much of the Advanced LIGO interferometers with a new detector operating at 120 K. The mirrors will be made out of 150 kg Silicon substrate. The mirrors, which will be suspended as a pendulum with silicon suspension fibers, will be housed inside an all enclosing shroud held at 77 K environment using liquid nitrogen. Both the suspension fibers and the mirror will be held at 120 K at pressure $10^{-6}$ Torr. Under these conditions, radiative transport is the dominant mode of heat transfer over convection or air conduction. The barrel of the cylindrical mirrors will be covered with high emissivity coating to facilitate heat loss by radiation. The feasibility of cooling a silicon mirror only by radiation and to maintain its temperature at 120 K is yet to be investigated. The purpose of this project is to investigate the radiative cooling of silicon mirrors for the Third Generation LIGO.

4.1 Experimental Interest

We are interested in finding the feasibility, improvement and drawbacks of using silicon mirror substrate at 120 K and of using high-emissivity coating. We are modeling the silicon mirror substrate with small silicon wafers. The wafer is placed in a 77 K environment inside a nitrogen dewar at pressure $10^{-6}$ Torr. The test will be done on the silicon wafers in two different states: i. uncoated bare wafer, and ii. wafer coated with high-emissivity coating. For each of these two states, two different physical properties are being measured: i. the emissivity, and ii. the quality factor $Q$ of the silicon wafer.
4.1.1 Emissivity of Wafer

The emissivity of a material is the relative ability of its surface to emit energy by radiation. It is the ratio of energy radiated by a particular material to energy radiated by a black body at the same temperature. A true black body have an emissivity of $\epsilon = 1$, while any real object have $\epsilon < 1$. Duller and blacker material has $\epsilon$ closer to 1, while highly polished reflective material has $\epsilon$ closer to 0. The emissivity of a material depends on temperature, wavelength and thickness of the material.

The silicon mirror being a reflective material will have low emissivity and will radiate energy very slowly. Since radiative transfer is the only cooling process available for the test mirrors, the process will be facilitated by the application of high-emissivity coating on the barrel of the cylindrical mirrors. The coating, which has $\epsilon$ closer to 1 will make the mirror a more effective radiator speeding up the cooling process.

We will be measuring the emissivity of our silicon wafer at 120 K without coating to reveal the effectiveness of the mirror as a radiator, and then with coating to find the change in emissivity. The wafer will be held at 120 K in a 77 K environment by a heater mounted on the wafer. Since, radiative transfer is the only cooling process*, the power input into the heater will be equal to the power lost by the wafer by radiation. The emissivity can be calculated from the measured power input using the Stefan-Boltzmann law. This is explained in greater detail in Ch. 5.

4.1.2 Mechanical Q of the Wafer

The quality factor Q of the wafer is inversely related to the loss in the system which can be due to structural damping, thermoelastic damping, etc. Due to silicon’s null coefficient of linear expansion at 120 K, it is expected to have no thermoelastic damping at this temperature, leading to a high-Q. We will measure the mechanical Q of uncoated bare silicon wafer at 120 K to verify whether the improvement in Q. However, the application of high-emissivity coating on the mirror wall will increase the level of structural damping, lowering the Q. The measurement of Q of the coated wafer will show the change the Q due to application of coating.

The mechanical Q of the wafer will be measured by a ring-down technique. The wafer will be excited to a resonant vibrational mode by driving it in a sinusoidal electric field. The Q can be calculated by measuring the free-decaying time of the driven amplitude. The process is explained in detail in Ch. 6.

4.2 Experimental Setup

The experiment is conducted inside a nitrogen dewar with the wafer being suspended from the cold plate. The suspension system for the wafer consists of an aluminum ring which is attached to the cold plate with insulating spacers. Multiple resistor heaters and silicon diode thermometers are mounted on the silicon wafer and aluminum ring for the emissivity measurement. The power going into the heaters are controlled externally to maintain the temperature of each. For the Q-measurement, two electrodes are attached on the aluminum ring on either side of the wafer. These are used for driving the wafer
at its resonant vibrational mode and also to sense the vertical motion of the wafer. These are used to measure the ring-down time of the wafer for Q calculation.

### 4.2.1 Silicon Wafer

The measurements are being carried out on 2 inch diameter 1250 µm thickness double side polished commercial n-type single-crystal prime CZ silicon wafer doped with phosphorus. The wafer has a linear cut across the edge as can be seen on Fig. 4.2.

The wafer is suspended with nylon fibers from the aluminum ring, and a small heating resistor and a silicon diode thermometer are attached on the wafer for controlling and monitoring its temperature. Mechanical contact to the surface of the wafer introduces structural damping which will lower the Q of the wafer and also change its vibrational mode. In order to avoid this, we chose a single vibrational mode to be used for the Q-measurement and found the nodal line for this mode. For resonance of a two-dimensional surface, the nodal line is the line on the surface where the surface remains motionless, dividing the entire surface into two separate regions vibrating with opposite phase. Since anything on the nodal line does not move during the excitation process, mechanical contact to the surface on the nodal line will

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**Figure 4.2: Silicon Wafer**

![Silicon Wafer](image)

**Figure 4.3: Vibrational mode of the Wafer**

![Vibrational mode of the Wafer](image)
introduce least amount of structural damping into the system. A model of this vibrational mode done in COMSOL is shown in Fig. [4.3] The nodal line can be seen as the dark blue region, which has undergone least displacement.

We drilled six holes across the nodal line as can be seen in Fig. [4.3] Three of these holes are used for suspending the wafer. A small chip resistor and a silicon diode thermometer are glued on next to two other holes. For the Q-measurement the wafer has to be electrically charged. A thin wire is soldered onto the wafer using indium solder (97% In, 3% Ag). Traditional soldering technique does not work on bare silicon, which has a thin-layer of oxide deposited on it. To prepare surface for soldering, I heated the wafer on a heating plate to about $300^\circ$ and then cooled it to about $200^\circ$ and then used an ultrasonic soldering iron to promote wetting of the surface.

4.2.2 Dewar

The experiment is being carried out inside a double-tank nitrogen-helium dewar, with both tanks filled with nitrogen. The dewar consists of an outer aluminum shroud with removable top and bottom cover plates. The tanks are attached to the upper cover and can be taken out of the shroud. A valve is connected near the top of the shroud to which we have attached a turbo pump. The dewar was originally an optical dewar with an optical window near the bottom, which is now replaced with a new fitting for connecting a pressure gauge. There is a 3.2 cm gap between the cold plate and the bottom cover which we have used as our experimental space for suspending the wafer.

Refurbishing the Dewar

The optical helium dewar recovered by Rai is a few decades old but seemed to be in promising condition. It was disassembled completely for thorough cleaning and leak-testing. After some of the bigger leaks were sealed the dewar was tested with Helium Leak Detector ans was passed as leak-free. Using turbo pump the dewar can be pumped down to a milliTorr within 5 hours, and can then be cryo-pumped to a microTorr by filling with liquid nitrogen. This relatively long pumping-down time can be attributed to the water molecules trapped inside the aluminum foil wrapped around the walls of the inner and outer shroud. Here is a brief description of a few adjustments made to the dewar:

- The attachments on the cold-plate from the previous experimental purpose were removed. The existing threaded holes on the cold-plate are used for attaching the suspension system of the wafer.
4.2.3 Aluminum Ring for Thermal Control and Suspension System
An aluminum ring of outer diameter 4" and inner diameter 3.5" is attached to the cold-plate of the dewar using insulating spacers. There are 12 lateral through-holes around the ring, which provide passage for wires coming out of the ring. The two capacitive plates used for the Q-measurement are screwed onto either side of ring. The ring has two main purposes: to suspend the wafer, and to prevent thermal conduction by the wires coming out of the wafer.

The silicon wafer is suspended from the ring by three nylon fibers which pass through three holes on the wafer at 120° to one another. The wires are clamped onto the ring with adjusting screws and heat sink tubes inside the holes of the ring provide firmer grip. There are small pieces of teflon tubes inside the wafer holes to prevent the wire from touching the surface of wafer.
CHAPTER 4. RADIATIVE COOLING OF SILICON WAFER

Five wires from the wafer, two from resistor and diode each and one for charging the wafer, are clamped in a similar fashion inside the through-holes of the aluminum ring. To prevent thermal conduction through the wires, the ring itself is held at the same temperature as the wafer. Two 300Ω resistors mounted on the ring heat up the ring, and an attached silicon diode thermometer is used to monitor its temperature. Heat sink compound on the wire wall and heat sink tube inside the holes are used to thermally ground the wires to the ring.

4.2.4 Capacitive Plates for Driving the Wafer

There are two capacitive plates attached to the ring on either side of the wafer, used for driving the wafer at its resonant frequency and to detect its motion. The electrodes are made by Rai out of steel mesh cut in circle and welded onto thin steel bars. Circular steel mesh mounted on steel bars for attaching to the aluminum ring ring. Steel meshes were chosen over steel plates to allow for better radiative transfer between the wafer surface and the cold-plate. The wafers are separated from the ring with insulating spacers to prevent electrical conduction.

4.2.5 Assembly and Preparation

The biggest portion of the project was spent in assembling the parts to start the experiment. Figure 4.12 gives a pictorial description of the assembly process. After everything is cleaned and assembled, the dewar is sealed and pumped down. The pressure reaches millTorr range in about 5 hours, after which nitrogen is pumped in and the pressure drops to microTorr. It takes a further 8 hours for the ring and the wafer to reach steady state temperature after which the experiment can be started.
4.3 High-Emissivity Coating

The coating we are using is a black optical coating from Acktar Advancef Coatings Ltd. We have chosen the model Ultra-Black\textsuperscript{®}. This coating has a thickness of 13-25 \( \mu \text{m} \) and is claimed to have 90 \% at 35 \( \mu \text{m} \) wavelength, which is the peak wavelength of the blackbody spectrum at 120 K. The coating is put on one side of the wafer in a 35\text{mm} \times 35\text{mm} square pattern. The coating has to be vacuum deposited on the surface and two of the wafers were sent to Acktar’s headquarter in Israel to be coated.
Chapter 5

Measurement of Emissivity of Silicon

The silicon substrate mirrors for the Third Generation LIGO will be cooled inside an all enclosing shroud under high vacuum ($10^{-6}$ Torr) in a 77 K environment. Radiative heat transfer is the only available cooling technique and modelling of the cooling process will require a knowledge of the emissivity of the silicon. Effective radiative cooling of the substrate requires that it has a high emissivity. Thorough search of literature has not yielded a reliable source of measurement for the emissivity of silicon at 120 K. A theoretical estimation was made by Rai Weiss by using the silicon infrared absorption weighted by the blackbody spectrum as a function of temperature [11], as shown in Fig. 5.1.

Figure 5.1: Silicon Thermal Emissivity Estimation [11]

Hence, the purpose of the first part of this experiment is to measure the emissivity of bare silicon at 120 K. In the second part we will measure the emissivity of a high-emissivity coating at 120 K to find the improvement in the radiative cooling due to the application of the coating.
5.1 Method of Measurement

The Stefan-Boltzmann law states that the power radiated per unit area by a black-body radiator at all frequencies is proportional to the fourth power of the absolute temperature and is given by,

\[ \frac{P}{A} = \sigma T^4 \]  

(5.1)

where, \( \sigma \) is the Stefan-Boltzmann constant, and \( A \) is the surface area of the object. For a hot object, other than an ideal radiator, radiating energy to the cooler surroundings, the power radiated is given by,

\[ P = \epsilon \sigma A (T_{\text{object}}^4 - T_{\text{surrounding}}^4) \]  

(5.2)

Here, emissivity \( \epsilon \) is the relative ability of the surface of an object to emit energy by radiation. An ideal radiator, or a true black-body has \( \epsilon = 1 \), while for all other real objects, \( \epsilon < 1 \).

We can determine the emissivity of the silicon wafer near 120 K by measuring the power required to hold the temperature of the silicon wafer at a particular temperature. The wafer will be in a 77 K environment and radiative heat transfer is assumed to be the only mode of power loss from the wafer. Thermal heat conduction by the copper wires will be avoided by holding the aluminum ring at the same temperature as the wafer. Hence, at equilibrium the power input to the resistor \( R_{\text{wafer}} \) is equal to the power dissipated by radiation to the surroundings:

\[ P_{\text{input}} = \epsilon \sigma A_{\text{wafer}} (T_{\text{wafer}}^4 - T_{\text{surrounding}}^4) \]  

(5.3)

By making careful measurement of the power required to hold the temperature of the wafer near steady 120 K, the emissivity can be calculated by,

\[ \epsilon_{\text{wafer}} = \frac{P_{\text{input}}}{\sigma A_{\text{wafer}} (T_{\text{wafer}}^4 - T_{\text{surrounding}}^4)} \]  

(5.4)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{\text{wafer}} )</td>
<td>120 K</td>
</tr>
<tr>
<td>( T_{\text{surrounding}} )</td>
<td>77 K</td>
</tr>
<tr>
<td>( A_{\text{wafer}} )</td>
<td>( 2.0 \times 10^{-8} m^2 )</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>( 5.67 \times 10^{-8} W m^{-2} K^{-4} )</td>
</tr>
</tbody>
</table>

Table 5.1: Parameters for emissivity calculation.

The above is a highly idealized version of the emissivity calculation with the following assumptions:

1. The surrounding of the wafer, i.e. both the radiation shield and the cold-plate, are at 77 K and there is no temperature gradient.

2. The temperature of the ring is at the same temperature as the wafer and there is no heat loss by thermal conduction through the copper wires.
3. The copper wires are thermally grounded to the ring and are not conducting heat from the outside world through the electrical feed-through.

However, in the actual experiment there were a number of departures from the ideal conditions which I will explain in the following section.

5.2 Experimental Setup

The experimental setup for the emissivity measurement consists of various resistor heaters and silicon diode temperature sensors mounted on the wafer, ring and the radiation shield.

- A small chip 50 Ω resistor and a silicon diode temperature sensor is glued on the nodal line of the wafer surface. The glue used is a good thermal conductor but electrical insulator at low temperature.
- Two 300 Ω resistors in series and a silicon diode are attached to the aluminum ring.
- A silicon diode is glued to the radiation shield.

The resistors on the wafer and the ring are connected to two variable power supplies. The voltage and current through the resistors are changed manually to change the power supplied to the resistors.

The temperature sensing diodes are connected in series to a non-inverting op-amp circuit as shown in Fig. 5.3 to allow constant and equal current through the three diodes. The voltage across each of the diodes are measured and are compared with the voltage-temperature calibration curve for the diodes to extrapolate the temperature being monitored.
Once the temperature of the wafer, ring and shroud have all reached their respective steady state temperature the experiment is started. Figure A.1 shows a typical cooling curve of the wafer, ring and radiative shield from room temperature to the steady state temperature reached after the dewar is filled with liquid nitrogen.

The experiment is started by supplying the necessary heat to the aluminum ring in order to hold it at the particular temperature at which the emissivity will be measured. Once the ring is at a steady state the wafer is heated by adjusting the power to the resistor until it also reaches steady state at the same temperature. The power going into the wafer is recorded along with the temperature values for the emissivity calculation.

5.3 Thermal Analysis

During the experiment it became evident that the experimental conditions were not similar to the idealized conditions described in Sec. 5.1. In particular, the following problems led to a revision of the model.

1. The radiative shield did not reach 77 K temperature. Hence, the top and bottom surface of the wafer were exposed to different temperatures.

2. The temperature of the ring could not be held at same temperature as the wafer. Hence, there was heat loss by thermal conduction through the copper wires.

3. The copper wires might not be properly thermally grounded to the ring leading to a heat input from the outside world.

These effects are summarized in Fig. 5.4.
Hence, the new heat transfer equation is:

\[ P_{RAD1} + P_{RAD2} + P_{COND} = P_{IN} \]  \hspace{1cm} (5.5)

\( P_{RAD1} \rightarrow \) radiative power loss between upper face of wafer and the cold-plate
\( P_{RAD2} \rightarrow \) radiative power loss between lower face of wafer and the radiative shield
\( P_{COND} \rightarrow \) conductive power loss between wafer and ring
\( P_{IN} \rightarrow \) power input through the resistor

Hence the emissivity can be calculated from:

\[ \epsilon A (2T_{Wafer}^4 - T_{Shield}^4 - (77K)^4) + C(T_{Wafer} - T_{Ring}) = P_{IN} \]  \hspace{1cm} (5.6)

where \( \epsilon \) is the emissivity of the silicon, \( A \) is the area of one surface of the wafer, and \( C \) is the total conductivity of all the wires between the ring and wafer.

We performed the experiment in two separate trials, with three runs at different temperatures during each trial. The results are shown below.

### 5.3.1 Trial 1

<table>
<thead>
<tr>
<th>( T_{Wafer} ) (K)</th>
<th>( T_{Ring} ) (K)</th>
<th>( T_{Shield} ) (K)</th>
<th>( P_{IN} ) (mW)</th>
<th>( \epsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>110</td>
<td>175</td>
<td>0</td>
<td>0.13</td>
</tr>
<tr>
<td>125</td>
<td>110</td>
<td>175</td>
<td>5</td>
<td>0.13</td>
</tr>
<tr>
<td>130</td>
<td>110</td>
<td>175</td>
<td>8</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Table 5.2: Data from the first experimental run to measure Emissivity

The measured value of the emissivity for this trial is 0.13. The uncertainty is ±0.03 determined from the uncertainly in power input and the assumptions made in model.

### 5.3.2 Trial 2

During this second trial the wafer was noticed to be at a higher temperature than the radiative shield and ring, even when there was no heat input from the resistor. To account for this, I defined an added heating term \( P_{RT} \) defined as,

\[ P_{RT} = G(T_{RT} - T_{Wafer}) \]  \hspace{1cm} (5.7)

<table>
<thead>
<tr>
<th>( T_{Wafer} ) (K)</th>
<th>( T_{Ring} ) (K)</th>
<th>( T_{Shield} ) (K)</th>
<th>( P_{IN} ) (mW)</th>
<th>( \epsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>110</td>
<td>175</td>
<td>0</td>
<td>–</td>
</tr>
<tr>
<td>125</td>
<td>110</td>
<td>175</td>
<td>5</td>
<td>0.08</td>
</tr>
<tr>
<td>130</td>
<td>110</td>
<td>175</td>
<td>8</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Table 5.3: Data from the second experimental run to measure Emissivity

where, \( T_{RT} \) is the room temperature or 295 K, and \( G \) is the thermal conductivity from outside the dewar which is unknown. Using the data, \( G \) was solved for and used to calculate \( \epsilon \).
The measured value of the emissivity for this trial is 0.08. The uncertainty is ±0.03 determined from the uncertainty in power input and the assumptions made in model.

5.3.3 Measured Emissivity of Uncoated Silicon Wafer

Although neither of the two trials were performed under ideal conditions, the value for emissivity measured from the revised model are reasonably consistent between the different runs as seen in Fig. 5.5. This also agrees with the theoretical estimation [11]. A further trial will be performed on the uncoated wafer, before starting the emissivity measurement for the coated wafer.
Chapter 6

Measurement of Mechanical Q of Silicon Wafer

In the LIGO detectors, motions resulting from thermally induced fluctuations in the dimensions of coating and substrate material is a serious noise contribution within the detection band. As noted in Ch. 3, the linear expansion coefficient of silicon becomes 0 at around 120 K, resulting in an improvement in the mechanical Q for the material. A recent work at the Moscow University has verified this to be true for silicon wafers as shown in Fig. 6.1. The purpose of the first part of our experiment is to verify this result for uncoated silicon wafer. The second part will be repeat the experiment for wafers coated with high-emissivity coating. An external coating worsens the mechanical Q of a test mass by adding structural damping. We’ll determine the change in the mechanical Q of the mirror as a result of the application of an external coating.

6.1 Experimental Procedure for Determining Q

The mechanical Q-factor is determined by a ring-down technique. The substrate is excited in a particular resonant vibrational mode by means of an sinusoidal electrical field oscillating at the resonant frequency
CHAPTER 6. MEASUREMENT OF MECHANICAL Q OF SILICON WAFER

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Figure 6.2: Measuring Q from ring-down time.

of the substrate. After reaching a sufficiently high amplitude $A_0$, the excitation is switched off and the subsequent ring-down is recorded. The free-decaying amplitude $A(t)$ typically follows the exponential form,

$$A(t) = A_0 e^{-t/\tau}$$

(6.1)

where $\tau$ is the characteristic ring-down time or the time constant. This corresponds to the time taken for the amplitude to decrease by $1/e$ of the maximum value. The Q-factor can then be determined from

$$Q = \pi f_0 \tau$$

(6.2)

where $f_0$ is the resonant frequency of the wafer at which it is being driven.

6.2 Vibrational Mode and Nodal Line

As described in Sec. 4.2.1, it is important to find the nodal line of the wafer, i.e. the line on the surface which undergoes zero displacement while resonating at the vibrational mode. We chose to excite a simple vibrational mode of the wafer with a circular nodal line on which the holes for the suspension system will be drilled. Assuming the wafer to be completely circular, I calculated the resonant frequency of the chosen mode using,

$$f_{ij} = \frac{\lambda_{i,j}^2 h}{2\pi a^2 \left[ \frac{E}{12\rho(1-\nu^2)} \right]^{\frac{1}{2}}}$$

(6.3)

where $\lambda_{i,j}$ is (), $a$ is the radius, $E$ is the Young’s modulus, $\nu$ is the Poisson’s ratio, $\rho$ is the density and $h$ is the thickness of the wafer [3]. I calculated the frequency of the (0,1) mode to be 7.3 kHz, using $\lambda_{0,1} = 9.084$ and a Poisson ratio of $\nu = 0.218$. The ratio $r/a$ for this vibrational mode is 0.681, where $r$ is the radius of the nodal line. The resonant frequency and nodal line calculation was verified by modeling the wafer in COMSOL to find the eigenfrequency. COMSOL is a design and finite element analysis software for modeling and simulating physics-based problems. I used this package to simulate the vibrational modes of the wafer as shown in Fig. 6.3.
6.3 Experimental Setup for Determining Q

The experiment for measuring the mechanical Q of the wafer is being carried on inside the nitrogen dewar with the same suspension setup as described previously. The capacitive plates mounted on the aluminum ring are used as driving plates to drive the wafer at its resonant frequency and as position sensor to monitor the position of the wafer to detect its vibrational mode. A thin wire soldered on the nodal line of the wafer is used to electrically charge the wafer. The experiment is started at room temperature and pressure to detect the resonant frequency of the desired vibrational mode of the wafer. Next the dewar is pumped down to a pressure of $10^{-7}$ Torr and then filled with liquid nitrogen. The heater resistor mounted on the silicon wafer is used to vary its temperature. The experiment to determine mechanical Q is repeated over a range of temperatures near 120 K to find the relation between temperature and Q for silicon.

6.3.1 The Mixer Circuit

A 50 kHz AC signal, $V_{\text{sense}}$, for position sensing and an AC signal, $V_{\text{Drive}}$, tuned to the resonant frequency of the wafer are applied to the capacitive plates after passing through the mixer circuit as shown in Fig. 6.5. Capacitors and resistors are added to isolate the two signals from each other. The values of the capacitors and resistors are chosen to satisfy the following conditions:

$$\frac{1}{RC} \gg 2\pi(1kHz)$$

$$\frac{1}{RC} \ll 2\pi(100kHz)$$

(6.4)
These criteria ensure that the impedance of the Sensor circuit is high for $V_{\text{Drive}}$, and the impedance of Driver circuit is high for $V_{\text{sense}}$. The signal generated on the wafer, $V_{\text{wafer}}$ is amplified by a lock-in amplifier and is then monitored with an oscilloscope and a spectrum analyzer.

### 6.3.2 Position Sensing

The small movement due to the flexure of the wafer when it is driven at its resonant frequency is detected using capacitive technique. The wafer is electrically charged by the application of a DC voltage. If the wafer is located a distance $x$ from the center of the two plates, a time-dependent voltage $V_{\text{wafer}}$ is generated on the wafer given by:

$$V_{\text{wafer}} \propto \frac{x}{d} V_{\text{sense}}$$  \hspace{1cm} (6.5)

where $d$ is half the distance between the two plates. When the wafer is closer to the top-plate, $x > 0$ and $V_{\text{wafer}}$ is in phase with $V_{\text{sense}}$, and when it is closer to the bottom plate, $x < 0$ and $V_{\text{wafer}}$ is out-of-phase with $V_{\text{sense}}$.

A lock-in amplifier is used to amplify $V_{\text{wafer}}$ which has a high signal-to-noise ratio. Lock-in-amplifier is a phase-sensitive detector which can extract signal with known frequency from an extremely noisy environment. $V_{\text{wafer}}$ is the input to the lock-in-amplifier while $V_{\text{sense}}$ is used as the the reference signal both of which are at 50 kHz. Inside the lock-in, $V_{\text{sense}}$ is multiplied with $V_{\text{wafer}}$. If the wafer is static, the output of the lock-in is:

$$V_{\text{LI}} = V_{\text{wafer}} \sin(2\pi f_0 t + \theta_{\text{wafer}}) \ast V_{\text{sense}} \sin(2\pi f_0 t + \theta_{\text{sense}})$$

$$= \frac{1}{2} V_{\text{wafer}} V_{\text{sense}} \cos(\theta_{\text{sense}} - \theta_{\text{wafer}}) - \frac{1}{2} V_{\text{wafer}} V_{\text{sense}} \cos(4\pi f_0 t + \theta_{\text{sense}} + \theta_{\text{wafer}})$$  \hspace{1cm} (6.6)
The output from the lock-in has two frequency components, one at the difference frequency and one at the sum frequency of the reference and input signal. Since both these signals are at the same frequency, the difference frequency is zero and we get a DC signal as an output. The AC signal at twice the frequency is removed through a low-pass filter inside the lock-in. The low-pass filter also integrates the random noise in the DC output to zero.

(a) Schematic of Position Sensor  
(b) Signal generated on wafer due to its position between the plates.

Figure 6.6: Position Sensing by Capacitive Plates

Since the position of the wafer cannot be manually adjusted finely enough to be at the exact center of the two capacitive plates, $V_{\text{wafer}}$ may not be equal to zero at the beginning of the experiment. This is problematic since the small $V_{\text{wafer}}$ due to unbalancing is amplified and overloads the lock-in. This sets a limit to which the gain $G$ can be set reducing the sensitivity of the motion sensor. To avoid this problem a trim resistor and a trim capacitor are connected to the transformer. These act as potential dividers varying the impedance between the two plates and hence dividing the voltage between them. When the plate is static, the trim resistor and capacitor are adjusted to balance both the real and imaginary part of the impedance, such that the output of the lock-in is lowest at the highest gain.

6.3.3 Driving Vibrational Mode

A driving voltage tuned to the resonant frequency excites the wafer to its vibrational mode. Once the wafer is excited to sufficient amplitude the driving voltage is switched off and the ring-down of the excitation is monitored on the oscilloscope. Very high driving voltage is required to excite the wafer and we are currently using a 100 V peak-to-peak drive. However, this may not be high enough to drive the flexure mode of the wafer. We have driven the longitudinal vibrational mode of the suspension system, and the results have indicated that we need a higher driving voltage to excite the flexure vibrational mode of the wafer.

6.4 Vibrational Mode of the Suspension System

In order to test the operation of our motion sensor and driver system, we electrically drove the longitudinal vibrational mode of the suspension system. Since this mode has a much lower Q, it is more easily detectable. A heavy stomp on the ground is enough to excite this mode and we observed the ring-down on the oscilloscope. The frequency of this mode is calculated to be 30 Hz and Q is 40. This mode was also excited electrically and peak at 30 Hz can be seen on the Spectrum Analyzer (Fig. 6.7).
6.5 Search for Vibrational Mode of Flexure of Wafer

Several attempts were taken at detecting the resonant frequency of the flexure vibrational mode of the wafer. The Q of this mode should be around $10^4$. Hence, for a 7 kHz resonant mode the FWHM will be about a 10th of a Hz. After sufficient improvements are made to the sensor and driver and the vibrational mode is detected at RTP, the dewar will be pumped down and cooled. Several Q measurements will be taken for a range of temperatures around 120 K. The same process will then be repeated for the coated wafer.
CHAPTER 6. MEASUREMENT OF MECHANICAL $Q$ OF SILICON WAFER

Figure 6.8: Setup for measuring $Q$. 
Appendix A

Typical Cooling Curve for the System

Figure A.1: Cooling Curve of Wafer, Ring and Shield
Appendix B

Mixer Circuit Board

Figure B.1: Schematic of the Printed Mixer Circuit Board drawn using EAGLE
Figure B.2: Board Diagram drawn using EAGLE

Figure B.3: The Mixer Circuit Board
Appendix C

Ackter High-Emissivity Coating

**Characteristics of Acktar Black™ Coatings (Typical)**
Coatings can be applied directly on discrete components and on roll material

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Nano Black®</th>
<th>Magic Black™</th>
<th>Vacuum Black™</th>
<th>Fractal Black™</th>
<th>Ultra Black™</th>
<th>Litho Black™</th>
<th>Metal Velvet™</th>
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</thead>
<tbody>
<tr>
<td>Coating thickness, µm</td>
<td>0.3 - 1</td>
<td>4 - 7</td>
<td>3 - 5</td>
<td>5 - 14</td>
<td>13 - 25</td>
<td>0.8 - 2</td>
<td>5 - 7</td>
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<td>Working temperature</td>
<td>-70°C to 250°C</td>
<td>-269°C to + 350°C (4%K to 623°K)</td>
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<tr>
<td>Weight of coating, mg/cm²</td>
<td>0.1 - 0.25</td>
<td>1.1 - 1.6</td>
<td>0.7 - 1.1</td>
<td>1.6 - 3.2</td>
<td>3.3 - 6.5</td>
<td>0.1 - 0.25</td>
<td>1.4 - 3.2</td>
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<tr>
<td>Abrasion resistance</td>
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<td>light</td>
<td>moderate</td>
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<td>moderate</td>
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<td>light</td>
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<tr>
<td>(ref MSL-C-460973)</td>
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<tr>
<td>Adhesion</td>
<td>Coated pieces withstand Scotch tape test (3M853 Crystal clear tape, strength of 13N per 25mm), without any evidence of coating removal.</td>
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<tr>
<td>Outgassing</td>
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<td>Chemical content</td>
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<td>Surface resistivity</td>
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<tr>
<td>Cleanability</td>
<td>Coated pieces withstand cleaning with ethanol, IPA or acetone with no change in optical and technical characteristics. Magic Black™, Metal Velvet™ and Litho Black™ should be cleaned very gently.</td>
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</table>

* Ultra Black can be provided on planar parts only

**Figure C.1: Characteristics of Acktar Black Coatings.**
Figure C.2: Reflectance curve of Acktar Ultr-Black, provided by Acktar.
Bibliography


